Introduction to granular suspensions

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Swirls of phytoplanktons in the Gulf of Finland
In Nature

Geophysics

Mud, rock, debris flows (rakaia river New Zealand)

Eruption of ashes (Anak Krakatau Indonesia)
Blood flow...

Cytoplasmic streaming...
In Industry

Construction materials (Mucem Marseille)

Paints...

Food industry...
Many disciplines:
- fluid/solid mechanics,
- soil mechanics,
- statistical physics

Still not well described... often with a phenomenological approach.

Difficulties:
- Very large number of particles
- Thermal fluctuations are absent
- Complex interactions...
Brownian/colloidal suspensions

Granular suspensions

Particle size

- Thermal agitation
- Repulsive forces (Electrostatic, Polymer coating)
- Adhesion forces (Van der Waals, ...)

Hydrodynamics
Solid contacts
Friction

$d (\mu m)$
Particle density

Sedimentation ($\rho_p > \rho_f$)

- Long range hydrodynamic interactions

Rheology ($\rho_p = \rho_f$)

- Geometry (solid contacts)
Particle density

Sedimentation ($\rho_p > \rho_f$)
- Long range hydrodynamic interactions

Rheology ($\rho_p = \rho_f$)
- Geometry, Friction
1 particle

\[ Re = \frac{\rho U_s d}{\eta_f} \rightarrow 0 \]

\[ \mathbf{D} = 6\pi \eta a \mathbf{U}_s \]

\[ \mathbf{F} = m \mathbf{g} \]

\[ \mathbf{u}(\mathbf{r}) = \frac{\mathbf{F}}{8\pi \mu} \left( \frac{1}{r} + \frac{\mathbf{r} \otimes \mathbf{r}}{r^3} \right) \]

(Stokeslet)

N-body with long range hydrodynamic interactions

Stokes 1851
N particles

Fluid motion:
\[ \nabla \cdot \mathbf{u} = 0 \, , \]
\[ -\nabla \cdot \rho + \mu \Delta \mathbf{u} + mg \sum_{m=1}^{N} \delta(\mathbf{r} - \mathbf{r}_m) = 0 \]

Point particles velocities:
\[ \dot{\mathbf{r}}_i = \mathbf{V}_s + \frac{F}{8\pi\mu} \cdot \sum_{j \neq i}^{N} \mathbf{T}(\mathbf{r}_{ij}) \]
\[ \dot{\mathbf{r}}_i^* = \frac{5}{8N} \sum_{j \neq i}^{N} \mathbf{T}(\mathbf{r}_{ij}^*) \cdot \mathbf{e}_z \]
\[ \mathbf{r}^* = \mathbf{r}/R \, , \, \dot{\mathbf{r}}_n^* = \dot{\mathbf{r}}_n/V_c \text{ with } V_c = \frac{NF}{5\pi\mu R} \, , \, \mathbf{T}(\mathbf{r}^*) = \frac{\delta_{ij}}{\mathbf{r}^*} + \frac{\mathbf{r}_i^* \mathbf{r}_j^*}{r^*3} \]

(S. Pic illustration of E. Guazzelli & J. Morris 2012)
\[ \mathbf{r}_i^* = \frac{5}{8N} \sum_{j \neq i}^{N} \mathbf{T}(\mathbf{r}_{ij}^*) \cdot \mathbf{e}_z \]
Sedimentation \( (\rho_p > \rho_f) \)

- dominated by **long range hydrodynamics interactions**
- First order description (Stokeslet) captures well the dynamics
- Coupling microstructure/flow \( \rightarrow \) complex behaviors
Particle density

**Sedimentation** ($\rho_p > \rho_f$)

- Long range hydrodynamic interactions
- Multi-body

**Rheology** ($\rho_p = \rho_f$)

- Geometry, Friction
- Jamming volume fraction
Rheology \((\rho_p = \rho_f)\)

Imposed volume fraction

\[ \tau = \eta_f \eta_s(\phi) \dot{\gamma} \]

\[ P_p = \eta_f \eta_n(\phi) \dot{\gamma} \]

\[ \eta_s(\phi) ? \]
Hydrodynamic approach (from the dilute limit)

\[ u(r) \sim \frac{1}{r^2} \]

Stresslet

\[ \eta_s(\phi) = 1 + \frac{5\phi}{2} \]
Stresslet dissipation

\[ \eta_s(\phi) = 1 + \frac{5\phi}{2} + 5\phi^2 \]
Pair interactions

Multibody interactions: no exact analytical calculation

(Einstein Ann. Phys. 1906)

(Batchelor JFM 1970)
Empirical laws

Viscosity is set by the distance to Jamming!

\[ \eta_s(\phi) \sim (\phi_c - \phi)^{-2} \]
Granular approach (from the jamming limit)

\[ \phi \text{ not imposed, but the particle pressure } P_p \]

\[ \tau = \mu(J)P_p \]

\[ \phi = \phi(J) \]

Imposed pressure

Frictional rheology

(GdR midi EPJ 2004)
Granular approach (from the jamming limit)

Imposed pressure rheometer

\[ \mu = \frac{\tau}{P_p} \]

\[ \phi(J) = \phi_c - B J^{0.5} \]

\[ \eta_s = \frac{\mu(J(\phi))}{J} \]

(Boyer, Guazzeli, Pouliquen PRL 2011)
Microscopic origin of the viscosity divergence

\[ P \sim \eta_f \eta_s(\phi) \dot{\gamma}^2 \sim (1 - \phi) \eta_f \dot{\gamma}_{\text{loc}}^2 \]

Power dissipation

\[ \eta_s(\phi) \sim \frac{\dot{\gamma}_{\text{loc}}^2}{\dot{\gamma}^2} = L^2(\phi) \]

Lever effect

Q: how velocity fluctuations are amplified as \( \phi \rightarrow \phi_c \)?
Floppy modes theory
(DeGiuli, Lerner, Wyart PRE 2015)

Geometry: particles cannot overlap!

\[ \mathcal{L}(\phi) \]

Increase of velocity fluctuations as \( \phi \rightarrow \phi_c \)

Scalings for frictionless spheres:
\[ \eta_s(\phi) \sim \mathcal{L}^2(\phi) \sim (\phi_c - \phi)^{-2.8} \]

Scalings + DEM for frictional spheres:
\[ \eta_s(\phi) \sim \mathcal{L}^2(\phi) \sim (\phi_c - \phi)^{-2} \]
Sedimentation ($\rho_p > \rho_f$)

- Long range hydrodynamic interactions

Rheology ($\rho_p = \rho_f$)

- Geometry (Solid contacts)
More recently... (on rheology)

Particle friction $\rightarrow$ Rheological flow rules
Flowing properties

Shear thickening
Flow instabilities
Hysteresis

(Seto PRL 2013, Wyart & Cates PRL 2014)
Clavaud PNAS 2017

(Darbois CommPhys 2020)

(Perrin PRX 2019)
What are shear thickening suspensions?
A puzzle?

Granular Suspension

\[ \eta_s \eta_f = f(\phi) \sim (\phi_c - \phi)^{-2} \]

\( \phi_{\mu_p > 0} = 0.58 \)

Rate independent!
Origin of shear thickening

**Key idea:** introduce a short range repulsive force

$$F_{\text{rep}} \rightarrow P_{\text{rep}} = \frac{F_{\text{rep}}}{d^2}$$

(Left) Low stress $P < P_{\text{rep}}$ (Frictionless)

(Right) Large stress $P > P_{\text{rep}}$ (Frictional)

Shear thickening = frictional transition

(Seto PRL 2013, Mari JoR 2014)
Why so dramatic?

$\phi_c$ depends on $\mu_p$!

$\phi_{c}^{\mu_p > 0} = 0.58$

$\phi_{c}^{\mu_p = 0} = 0.64$
Frictional transition model: (Wyart & Cates PRL 2014)

Order parameter:

\( f = 0 \rightarrow \text{Frictionless} \) when \( P < P_{\text{rep}} \)

\( f = 1 \rightarrow \text{Frictional} \) when \( P > P_{\text{rep}} \)

\[ \phi_c \left( \frac{P}{P_{\text{rep}}} \right) = \phi_c^{\nu_s=0} (1 - f) + \phi_c^{\nu_s=1} f \]

\[ \frac{\eta_s}{\eta_f} = \left( \phi_c \left( \frac{P}{P_{\text{rep}}} \right) - \phi \right)^{-2} \]

(Singh JoR 2018)

Activation of friction \( \rightarrow \) negatively sloped rheograms!
Experimental validation

Silica beads 25 microns + water + NaCl

Repulsive force:

\[ F_{\text{rep}} = F_0 \exp\left(-r/\lambda_D\right) \]

Debye length:

\[ \lambda_d = \frac{0.304}{\sqrt{[\text{NaCl}]} \text{ (nm)}} \]

Access to friction: pressure imposed

\[ \ell_r = 3.73 \pm 0.80 \text{ nm} \]

(Extracted text and diagrams from Clavaud PNAS 2017)
Repulsive force $\theta = 6^\circ$ frictionless grains!  
Repulsive force $\theta \approx 30^\circ$ frictional grains!
Experimental validation

Nano-metric features control the macroscopic response of the system!

(Clavaud PNAS 2017)
Experimental validation

Repulsive force → Frictionless state under low stress → Shear thickening

Validation Frictional Transition Model

(Clavaud PNAS 2017)
- Optimize modern concretes

\[ \text{CaCO}_3 + \text{H}_2\text{O} \]

\[ \phi = 0.56 \]

\[ \phi = 0.72 \]

\[ \eta_s \]

\[ \phi_{\mu > 0} \]

\[ \phi_{\mu = 0} \]

Collaboration: Chryso
ANR: ScienceFriction

(Richards Rheol. Acta 2020)
Control of the flow properties by controlling friction?

Microscopic friction → Macroscopic rheology
Microscopic friction → Macroscopic rheology

- Control of the flow properties by controlling friction?

(Lin PNAS 2016) (Isa PNAS 2018)
Go beyond rheology: hydrodynamics
Shear thickening suspension down an incline

Not understood, no flow rules!
(Balmforth Phys. Let. A 2005)
S-shaped $A = \frac{d\dot{\gamma}}{d\tau} < 0$

No inertia $Re \ll 1$

Force balance (depth averaged):

$$\tau_b = \rho gh \sin \theta$$

Basal stress  Gravity
S-shaped $A = \frac{d\dot{\gamma}}{d\tau} < 0$  

No inertia $Re \ll 1$

Force balance (depth averaged):

$$\tau_b = \rho gh \sin \theta - \rho gh \cos \theta \frac{\partial h}{\partial x}$$

- Basal stress
- Gravity
- Hydrostatic pressure
S-shaped $A = \frac{d\gamma}{d\tau} < 0$

No inertia $Re \ll 1$

**Force balance (depth averaged):**

$$\tau_b = \rho gh \sin \theta - \rho gh \cos \theta \frac{\partial h}{\partial x}$$

**Basal stress**  Gravity  **Hydrostatic pressure**

→ Unstable if $A = \frac{d\gamma}{d\tau} < 0$
Linear stability analysis (depth averaged)

(Mass conservation + Force balance without inertia)

Layer thickness perturbation:

\[
\frac{\partial h_1}{\partial t} + \tilde{c} \frac{\partial h_1}{\partial x} = \frac{A}{\tan \theta} \frac{\partial^2 h_1}{\partial x^2}
\]

with \( \tilde{c} = 2 + A \)

\[\rightarrow \text{Newtonian fluid} \quad A = \frac{d \dot{\gamma}}{d \tau} > 0 \quad \text{Stable}\]

\[\rightarrow \text{Shear thickening (S-shaped)} \quad A = \frac{d \dot{\gamma}}{d \tau} < 0 \quad \text{Unstable}
\]

The bump “anti-diffuses”!
\[ \frac{R_{e_c}}{R_{e_K}} \]

**UNSTABLE**

**STABLE**

\[ \phi_{DST} \]

\[ R_{e_K} = \frac{5}{6 \tan \theta} \]

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**New instability**

Coupling negatively sloped rheology/free surface

\[ R_{e_c} = \frac{3 \tau c^3 (\phi_c - \phi)^4}{4 \eta_s^2 \rho g^2 \sin^2 \theta} \quad c_c / u_0 = 2 \]
Shear thickening suspension down an incline

Frictional transition $\Rightarrow$ S-shaped flow curve $+$ Coupling with free surface deformation $\Rightarrow$ Kinematic waves unstable without inertia

*New class of unstable waves:* extend to velocity-weakening materials (granular, micelles, mud…flows)?

Non-linear description?
Hysteresis of the avalanche angle

$$\Delta \theta = \theta_{\text{start}} - \theta_{\text{stop}}$$

Hysteresis should vanish without inertia

Tac-Tac model
(Quartier PRE 2000)
Self-induced fluidization
(DeGiuli PNAS 2017)

Models based on inertia

$$St = \frac{\text{inertia viscosity}}{18\eta_f} = \frac{\sqrt{\phi\Delta pgd^3}}{18\eta_f}$$

(Courrech PRL 2013)

Hysteresis should vanish without inertia
Hysteresis of the avalanche angle

Frictionless particles

Frictional particles

No friction $\rightarrow$ No hysteresis

Hysteresis without inertia (St<<1)!
Key ingredient for hysteresis: friction not inertia!

(Perrin PRX 2019)
Link with rheology?

\[
\mu = \tan \theta
\]

\[
J = \frac{\eta_f \dot{\gamma}}{P} = \frac{\eta_f \dot{\theta} R^2}{P h^2}
\]
Link with rheology:

Frictionless: monotonic rheology

\[ \Delta \mu = \mu - \mu_c^0 \sim J^{0.37} \]

consistent with predictions of DeGiuli PRE 2015.

Frictional: non-monotonic rheology

\[ \Delta \mu = \mu - \mu_c^1 \sim (J - J_C)^{0.7} \]
Hysteresis of the avalanche angle

\[ \Delta \theta = \theta_{\text{start}} - \theta_{\text{stop}} \]

Inter-particle friction \quad \rightarrow \quad Rheology \quad \rightarrow \quad Flowing property

Origin of velocity weakening?

(Perrin PRX 2019)
**Microscopic**

Particle friction

- Shear thickening

Frictional transition

**Macroscopic**

Rheological flow rules

- Flow instabilities

Coupling

- S-shaped rheology/free surface

- Hysteresis without inertia

- Non-monotonic flow rules

Hysteresis

Credit: C. Clavaud, A. Bérut, H. Perrin, B. Darbois Texier, H. Lhuissier, Y. Forrer, M. Wyart
Pipe Flow

Constant flow rate: backward traveling frictional plug?