Introduction to granular suspensions

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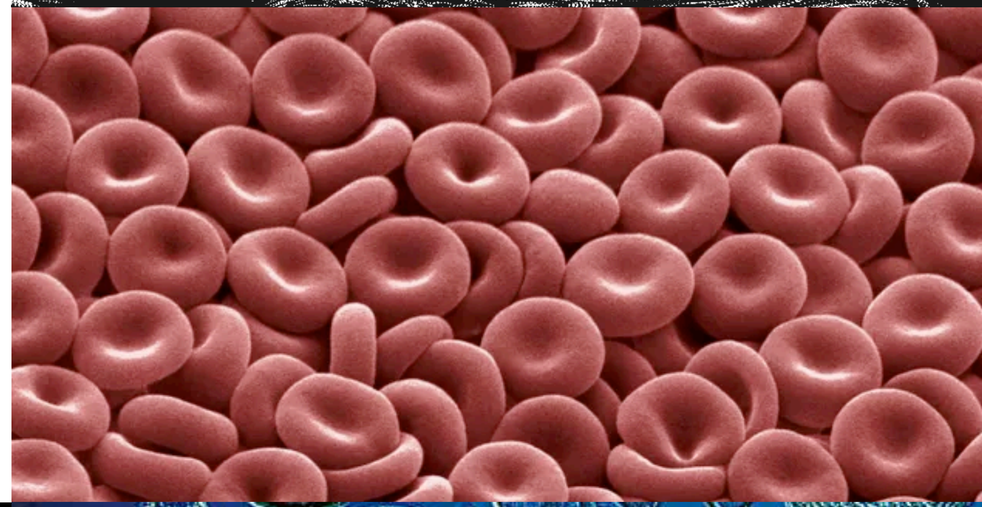
GDR MathGeoPHy - Oct. 2022



In Nature

Oceanography

Swirls of phytoplanktons in the Gulf of Finland

In Nature

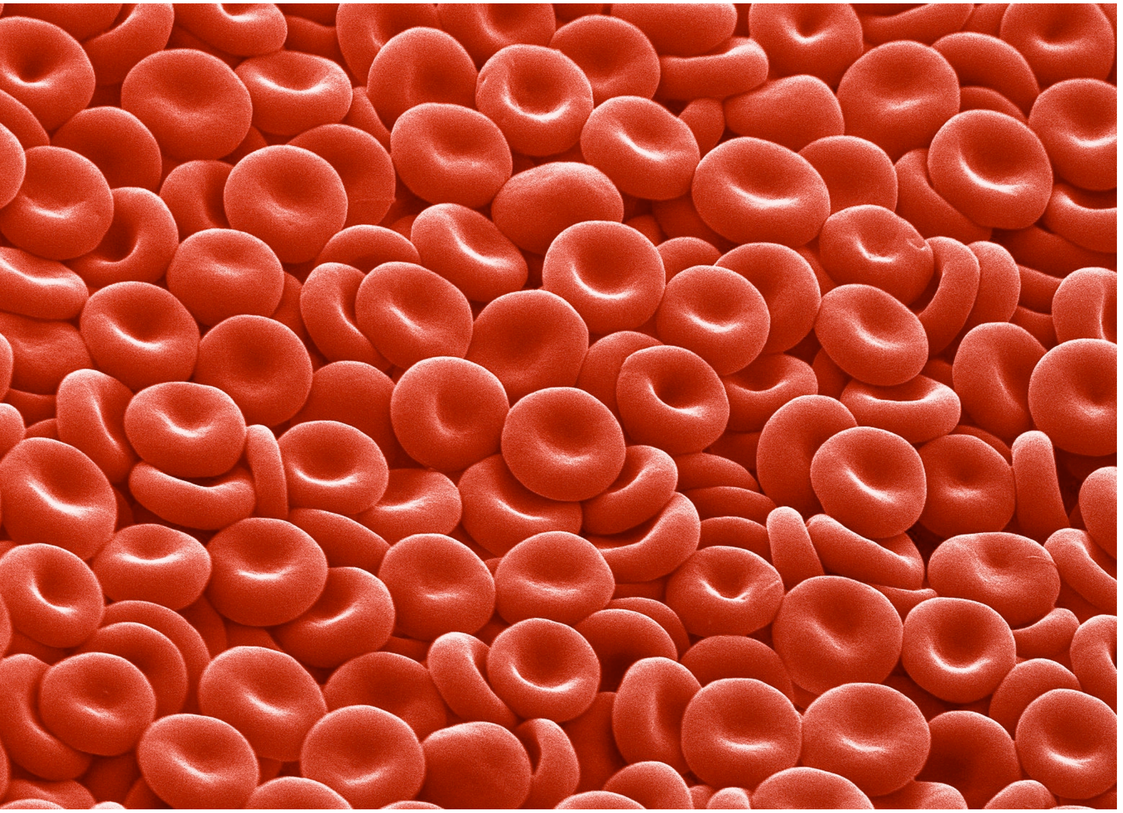
Geophysics

Mud, rock, debris flows (rakaia river New Zealand)

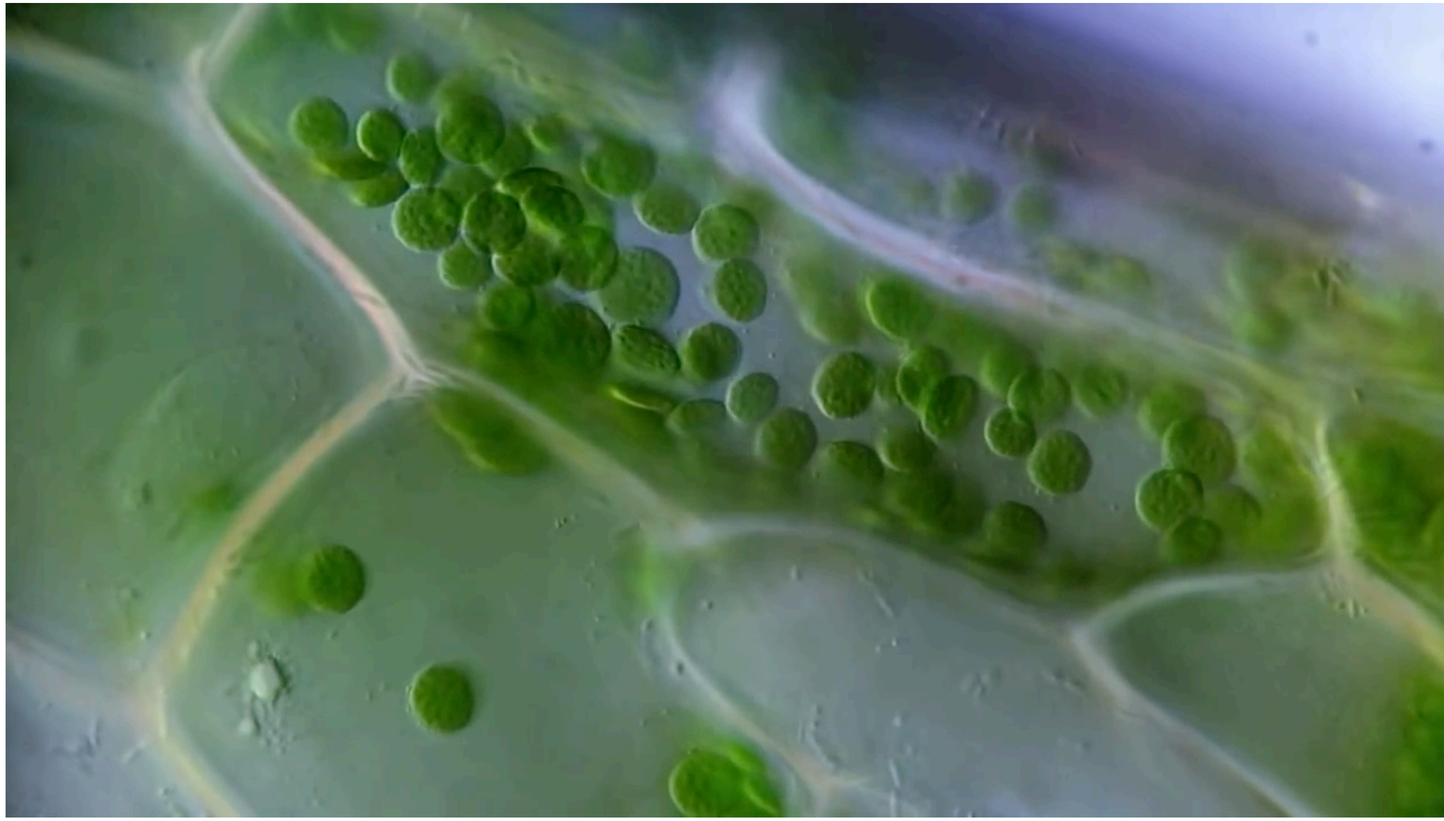
Eruption of ashes (Anak Krakatau Indonesia)

In Nature

Biology

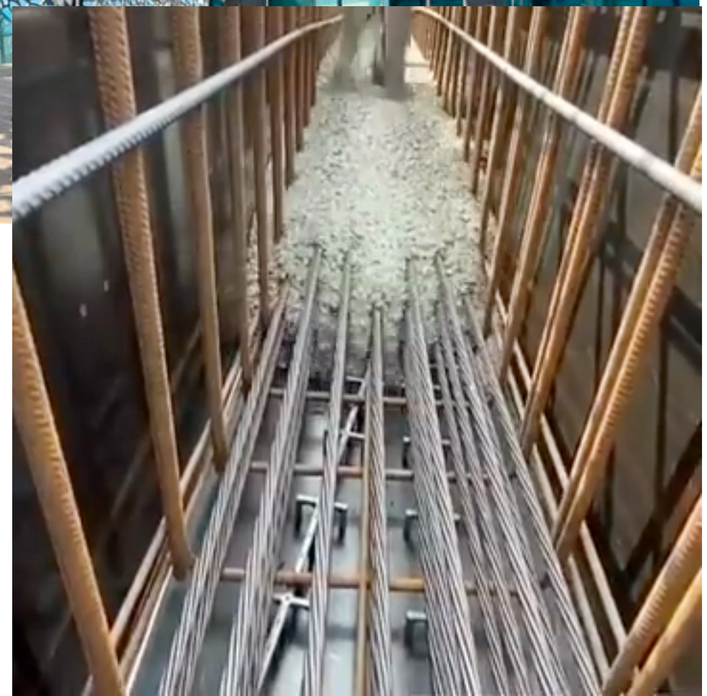


Blood flow…



Cytoplasmic streaming…

In Industry



Construction materials
(Mucem Marseille)

Paints…

Food industry…

Scientific interest

Many disciplines:

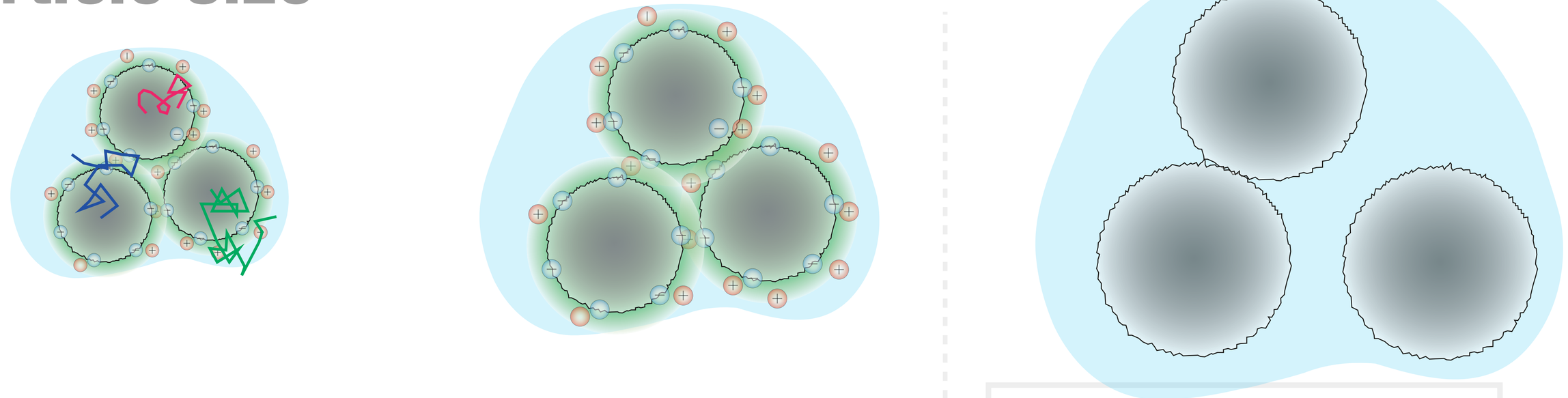
-fluid/solid mechanics,
-soil mechanics,
-statistical physics

Still not well described… often with a phenomenological approach.

Difficulties:

-Very large number of particles
-Thermal fluctuations are absent
-Complex interactions…

Particle size



Brownian/colloïdal suspensions

Granular suspensions

10^{-3} 1 10^{3} d (μm)

Thermal agitation

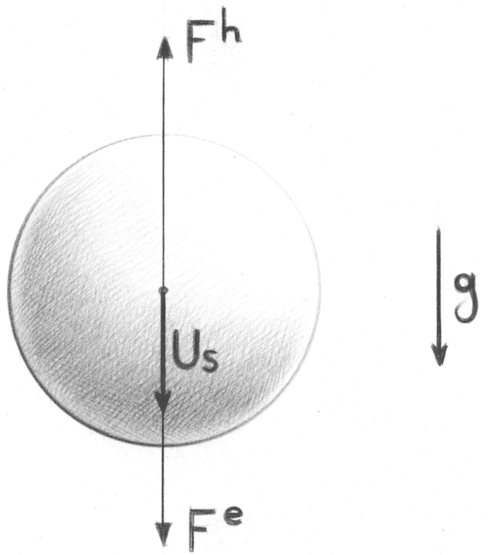
Repulsive forces (Electrostatic, Polymer coating)

Adhesion forces (Van der Waals,…)

Hydrodynamics
Solid contacts
Friction

Particle density

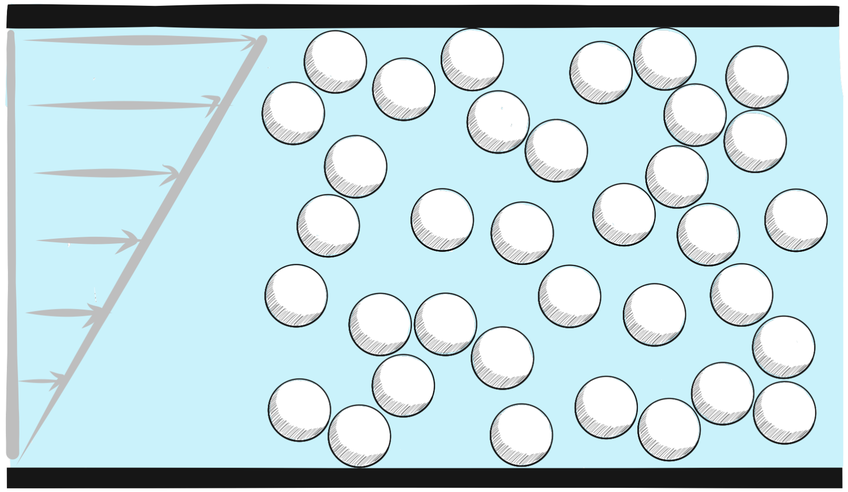
Sedimentation ($\rho_p > \rho_f$)



- Long range hydrodynamic interactions

$\neq$

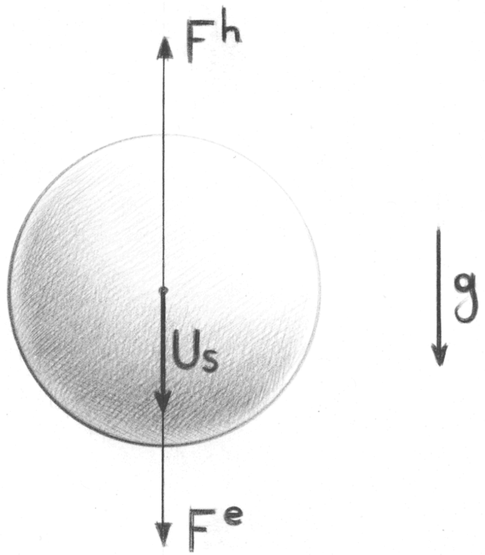
Rheology ($\rho_p = \rho_f$)



- Geometry (solid contacts)

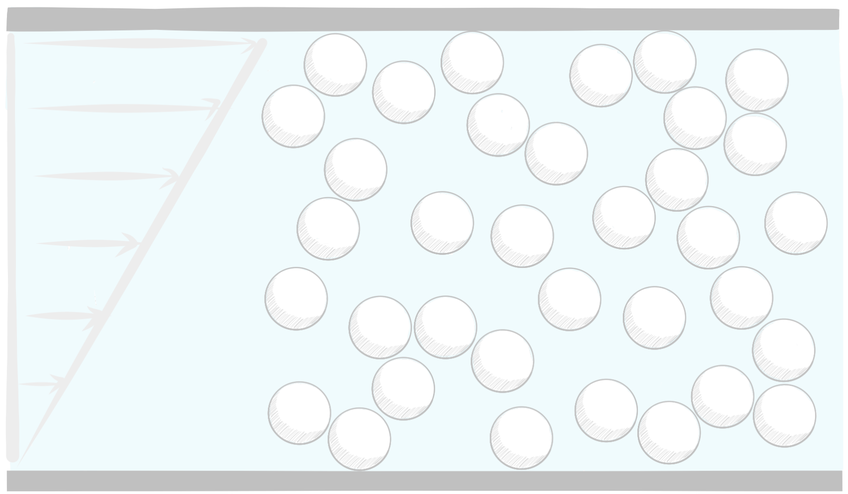
Particle density

Sedimentation ($\rho_p > \rho_f$)



- Long range hydrodynamic interactions

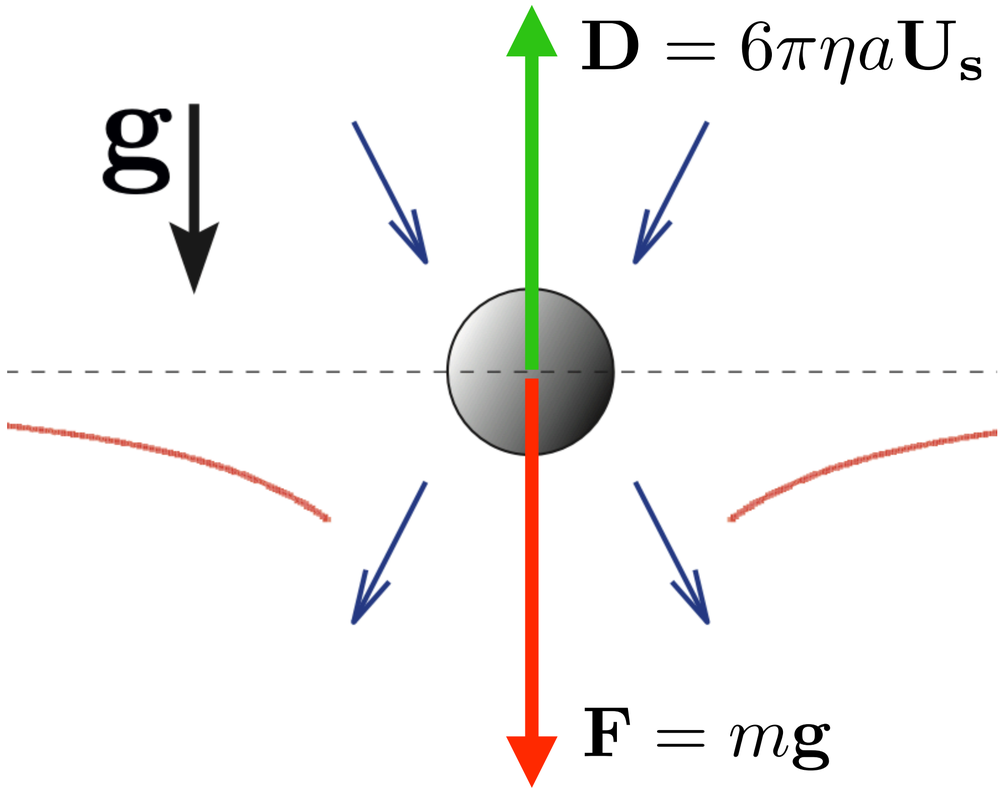
Rheology ($\rho_p = \rho_f$)



- Geometry, Friction

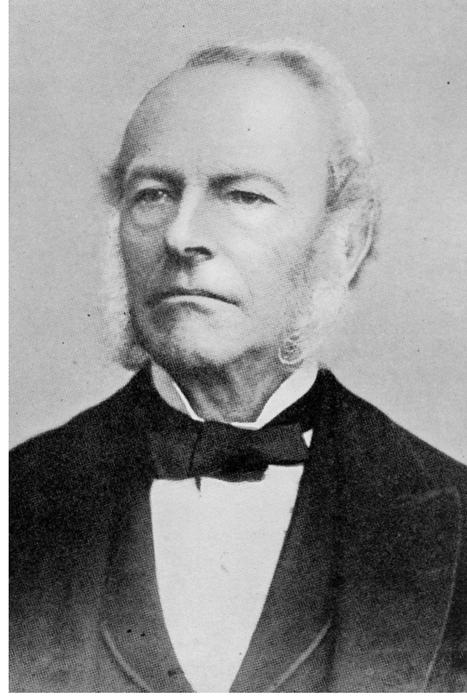
1 particle

$$Re = \frac{\rho U_s d}{\eta_f} \to 0$$



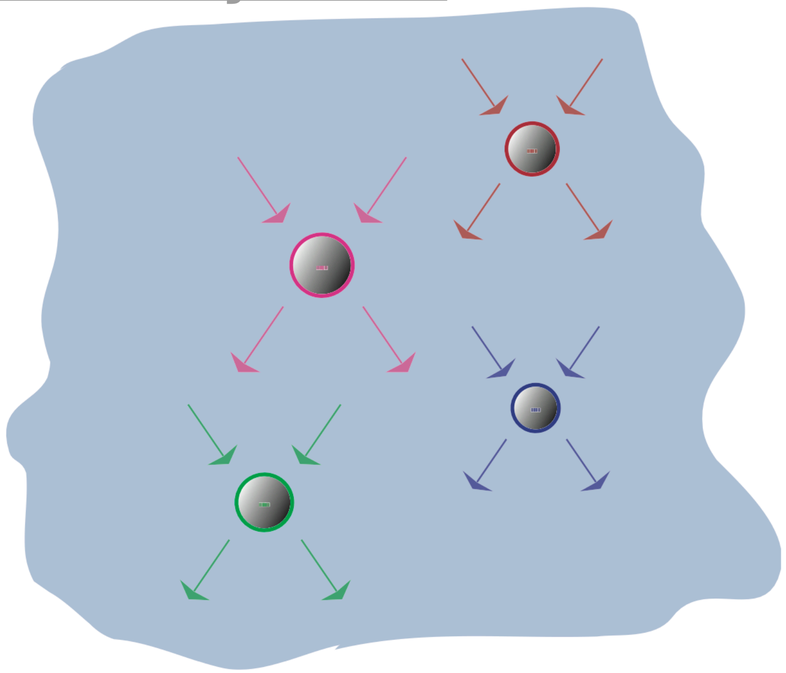
$$\mathbf{u}(\mathbf{r}) = \frac{\mathbf{F}}{8\pi\mu}\left(\frac{\mathbf{I}}{r} + \frac{\mathbf{r}\otimes\mathbf{r}}{r^3}\right)$$

(Stokeslet)



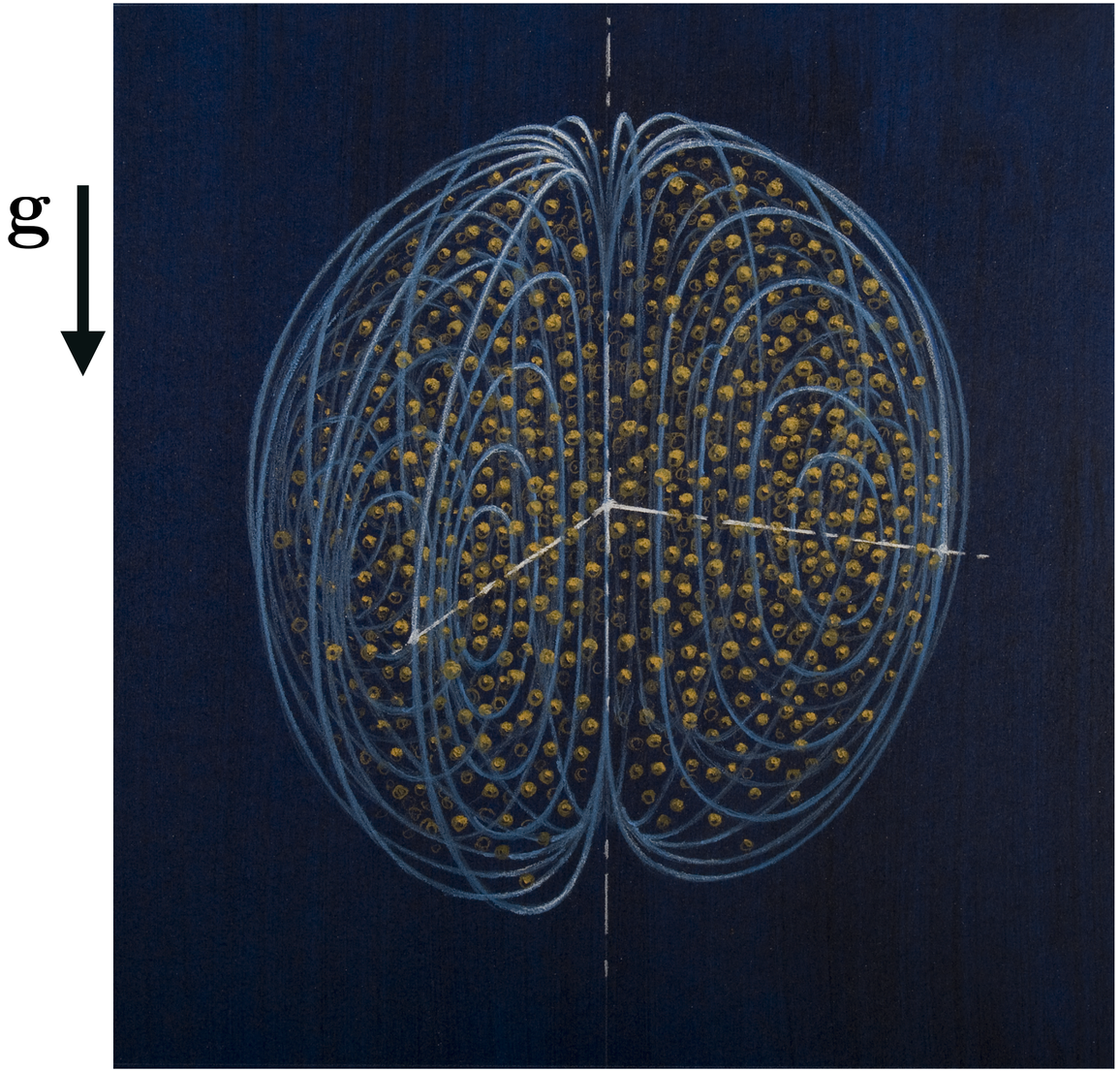
Stokes 1851

Collective dynamics



N-body with long range hydrodynamic interactions

N particles



(S. Pic illustration of E. Guazzelli & J. Morris 2012)

Fluid motion:

$$\nabla \cdot \mathbf{u} = 0 \ ,$$

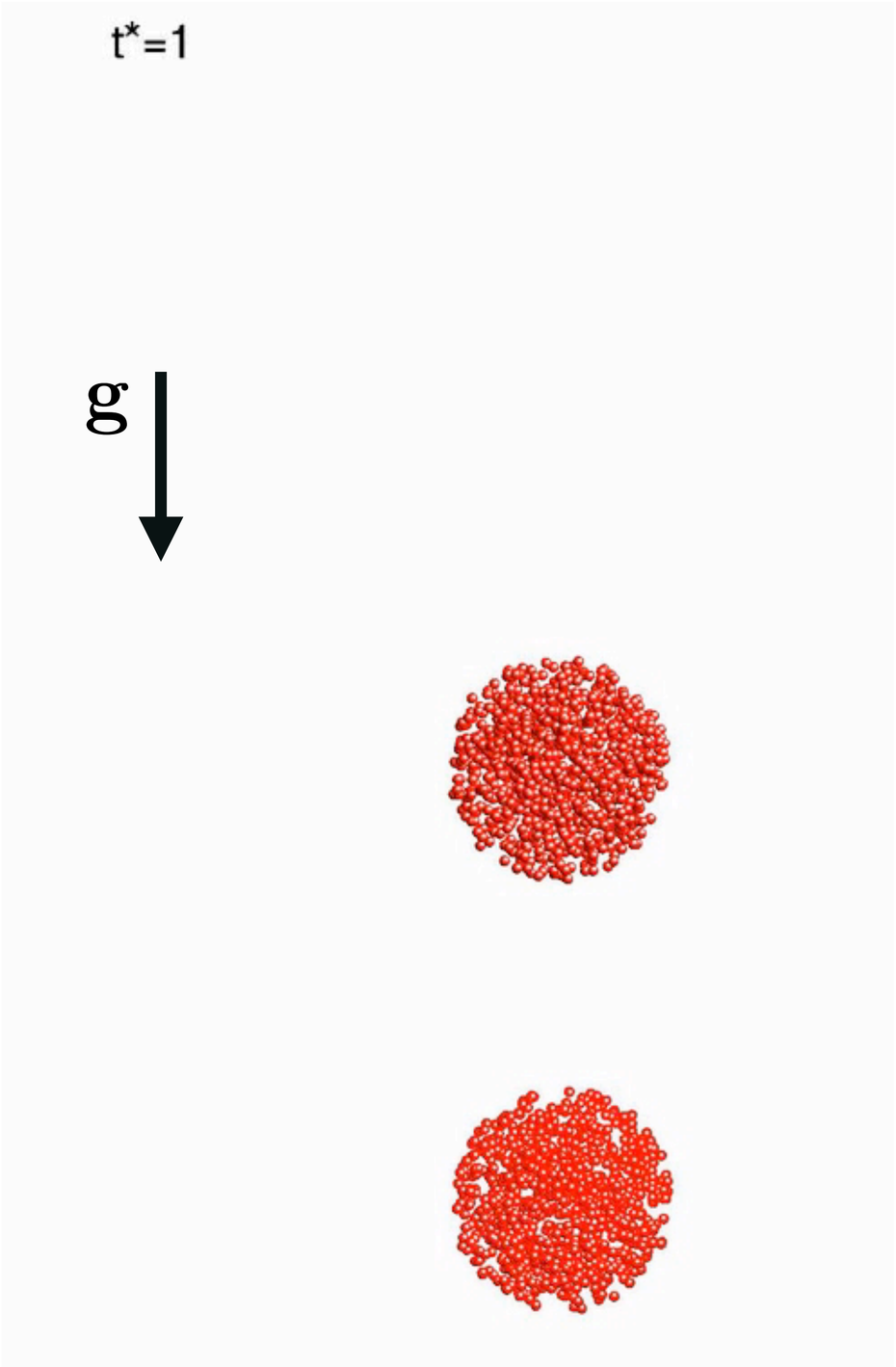
$$-\nabla \cdot p + \mu \triangle \mathbf{u} + m\mathbf{g} \sum_{m=1}^{N} \delta(\mathbf{r} - \mathbf{r}_m) = 0$$

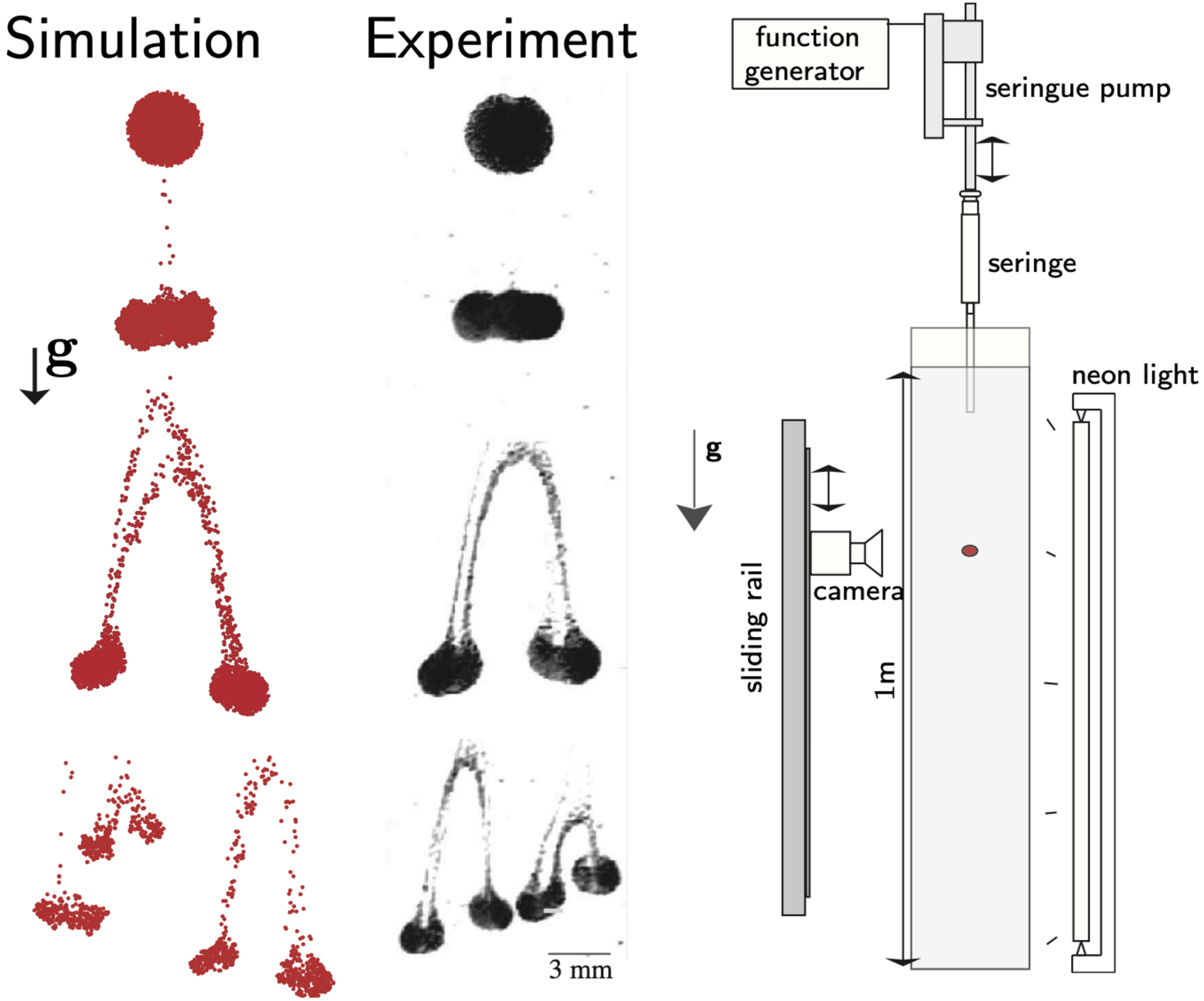
Point particles velocities:

$$\dot{\mathbf{r}}_i = \mathbf{V_s} + \frac{\mathbf{F}}{8\pi\mu} \cdot \sum_{j \neq i}^{N} \mathbf{T}(\mathbf{r}_{ij}) \rightarrow \boxed{\dot{\mathbf{r}}_i^* = \frac{5}{8N} \sum_{j \neq i}^{N} \mathbf{T}(\mathbf{r}_{ij}^*) \cdot \mathbf{e}_z}$$

$\mathbf{r}^* = \mathbf{r}/R$, $\dot{\mathbf{r}}_n^* = \dot{\mathbf{r}}_n / V_c$ with $V_c = \frac{NF}{5\pi\mu R}$, $\mathbf{T}(\mathbf{r}^*) = \frac{\delta_{ij}}{\mathbf{r}^*} + \frac{\mathbf{r_i^* r_j^*}}{r^{*3}}$

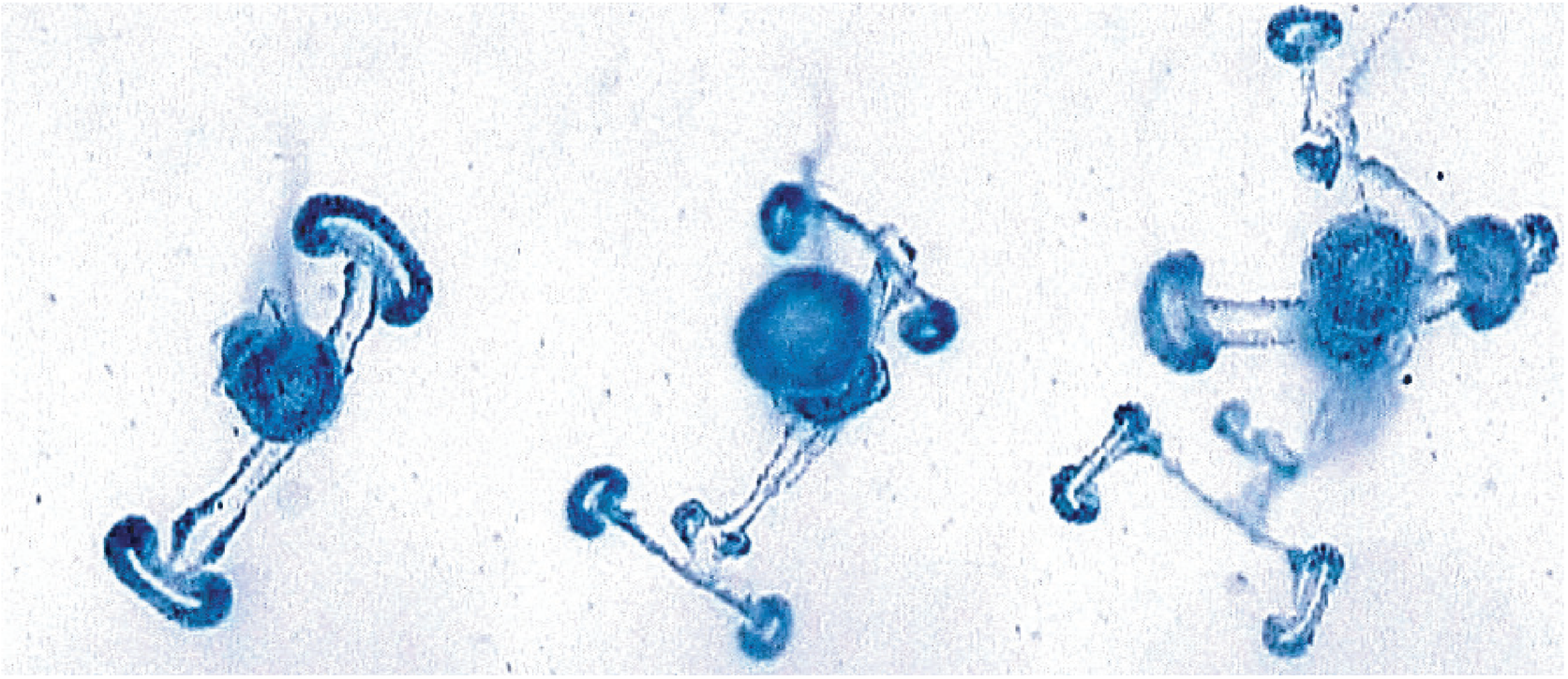
$$\dot{\mathbf{r}}_i^* = \frac{5}{8N} \sum_{j \neq i}^{N} \mathsf{T}(\mathbf{r}_{ij}^*) \cdot \mathbf{e}_z$$





(Metzger JFM 2006)

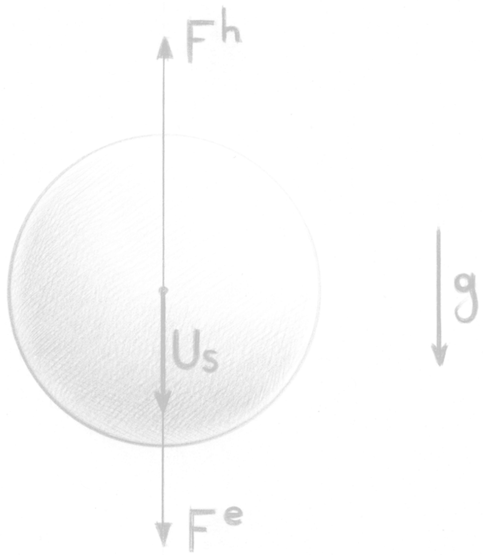
Sedimentation $(\rho_p > \rho_f)$



-dominated by **long range hydrodynamics interactions**
-First order description (Stokeslet) captures well the dynamics
-Coupling microstructure/flow → complex behaviors

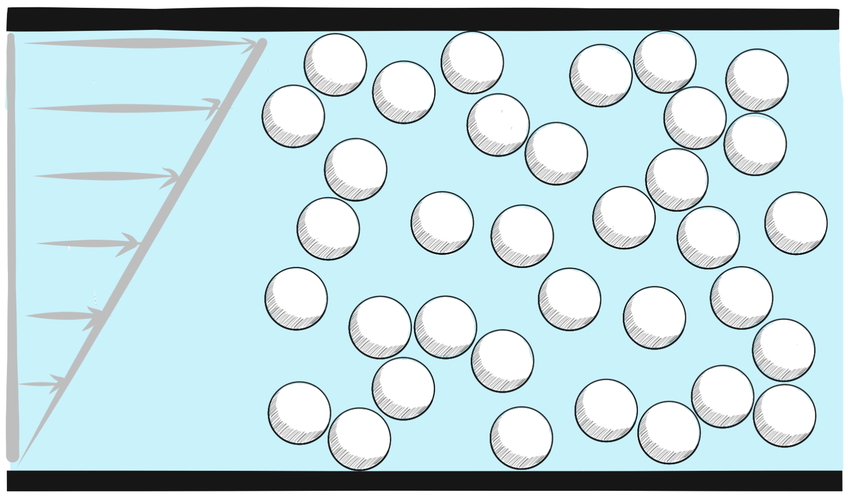
Particle density

Sedimentation ($\rho_p > \rho_f$)



- Long range hydrodynamic interactions
- Multi-body

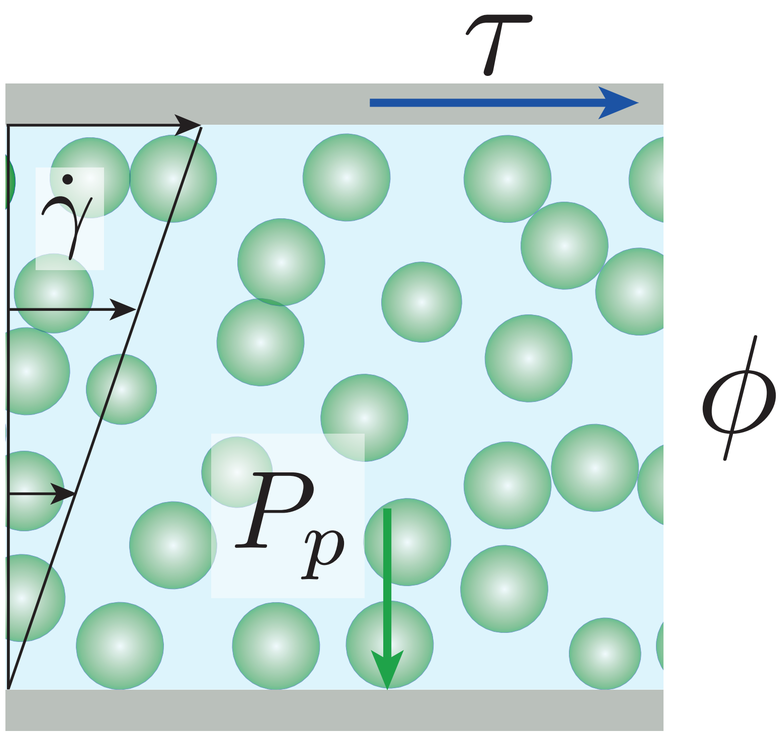
Rheology ($\rho_p = \rho_f$)



- Geometry, Friction
- Jamming volume fraction

Rheology $(\rho_p = \rho_f)$

Imposed volume fraction

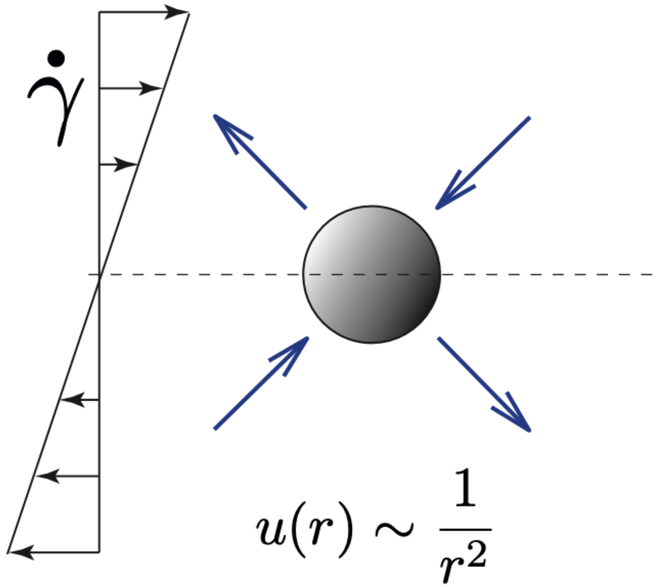


$$\tau = \eta_f \eta_s(\phi)\dot{\gamma}$$

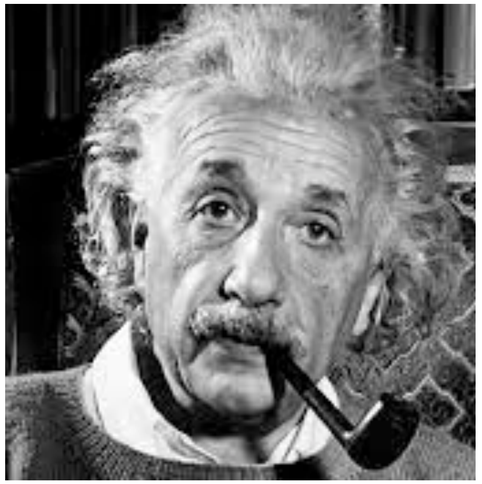
$$P_p = \eta_f \eta_n(\phi)\dot{\gamma}$$

$\longrightarrow$ $\eta_s(\phi)$?

Hydrodynamic approach (from the dilute limit)



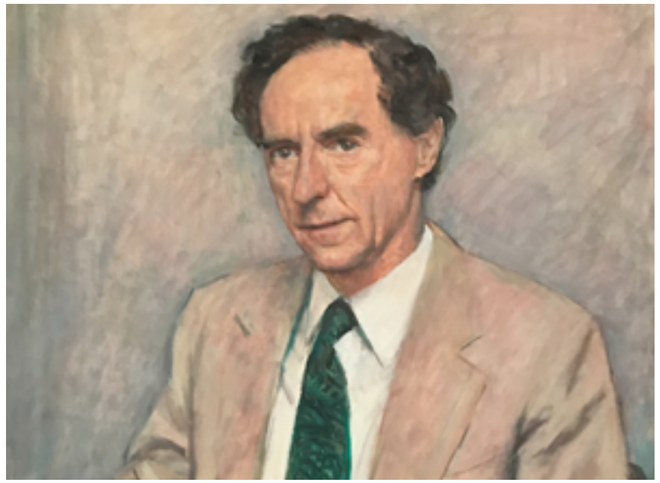
(Stresslet)



(Einstein Ann. Phys. 1906)

$$\eta_S(\phi) = 1 + 5\phi/2$$

Stresslet dissipation



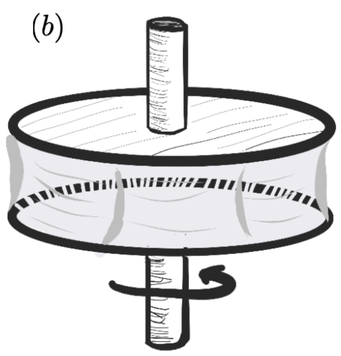
(Batchelor JFM 1970)

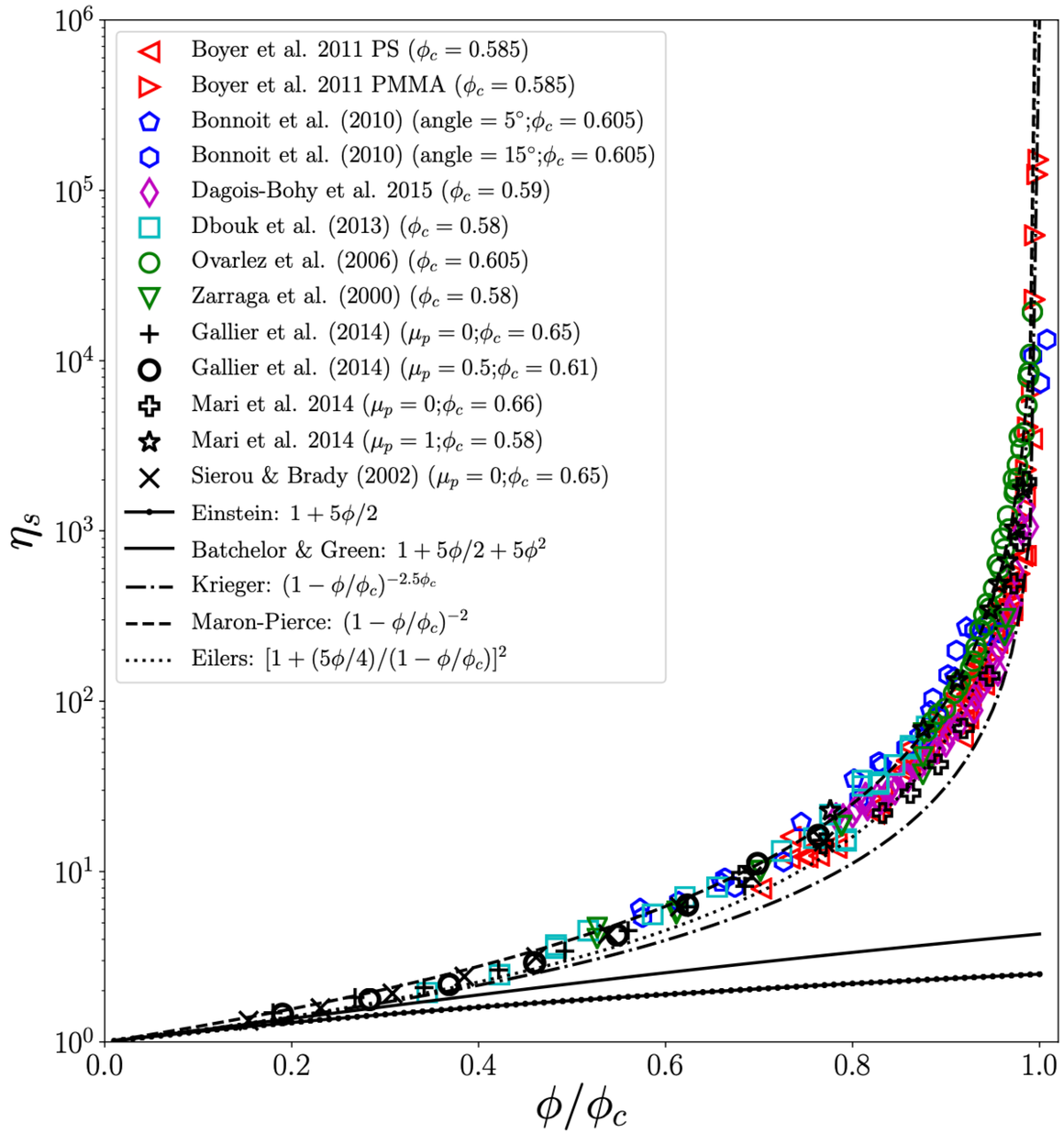
$$\eta_S(\phi) = 1 + 5\phi/2 + 5\phi^2$$

Pair interactions

→ **Multibody interactions: no exact analytical calculation**

Empirical laws



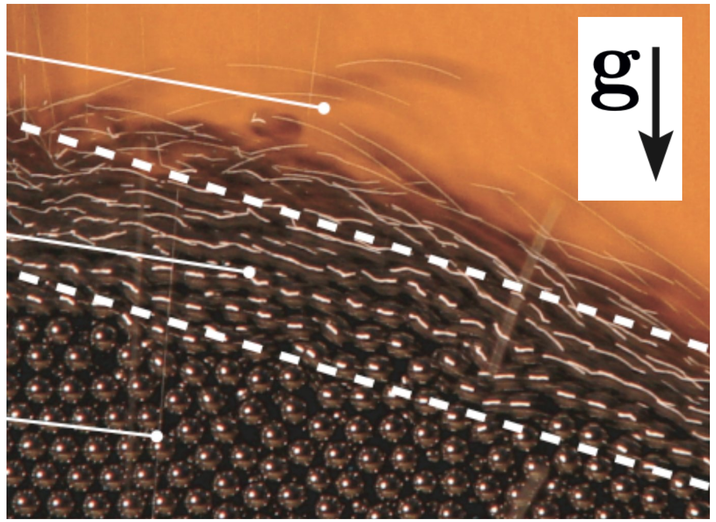


$$\eta_s(\phi) \sim (\phi_c - \phi)^{-2}$$

Viscosity is set by the distance to Jamming!

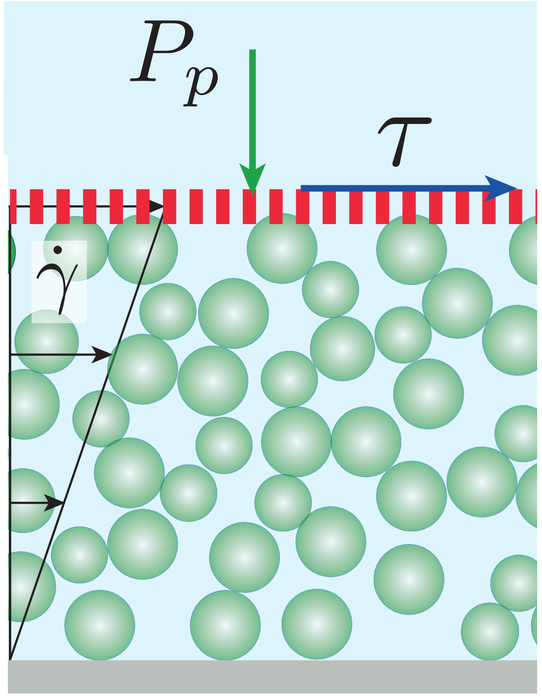
(Guazzelli JFM 2018)

Granular approach (from the jamming limit)



ϕ not imposed, but the particle pressure P_p

Imposed pressure



$$J = \frac{\eta_f \dot{\gamma}}{P_p}$$

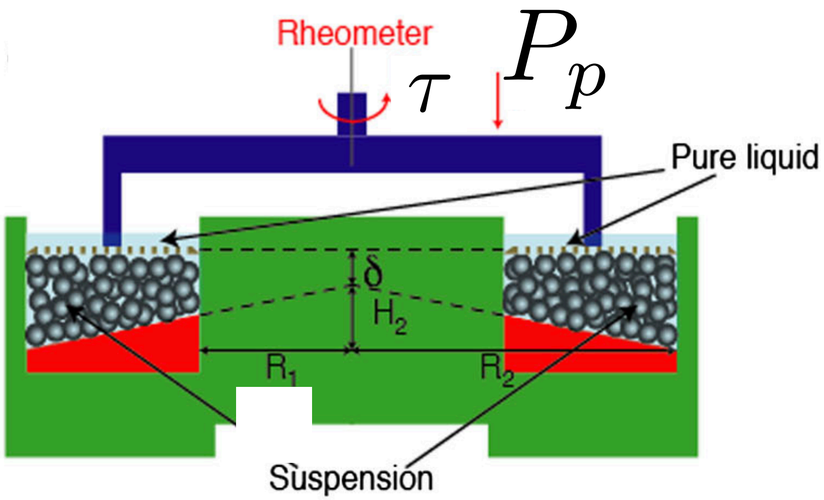
$$\tau = \mu(J) P_p$$

$$\phi = \phi(J)$$

Frictional rheology

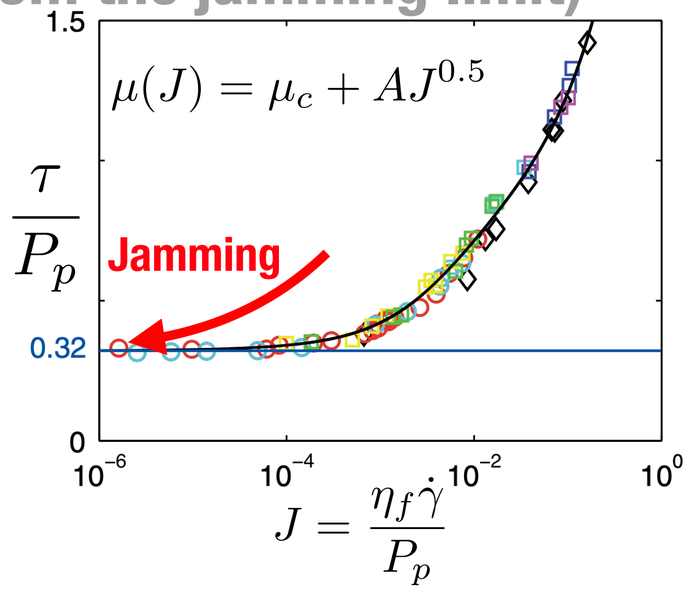
(GdR midi EPJ 2004)

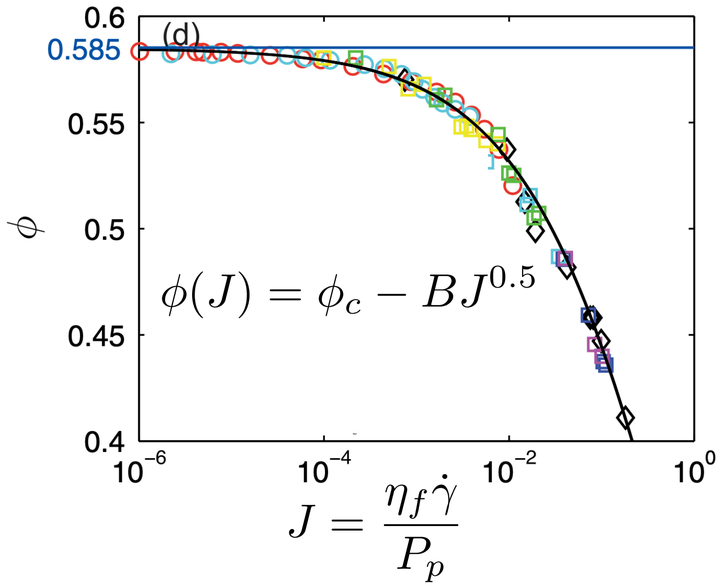
Granular approach (from the jamming limit)



Imposed pressure rheometer

$$\mu = \frac{\tau}{P_p}$$



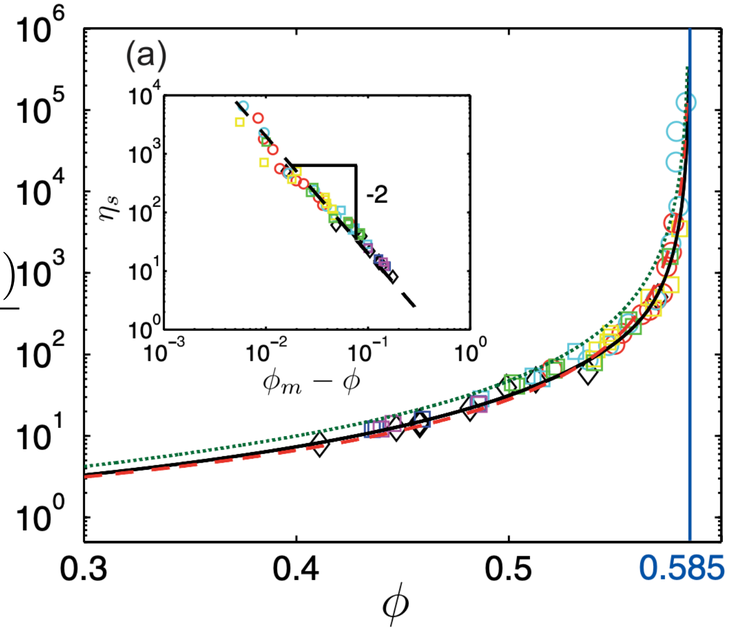


Imposed volume fraction

↕

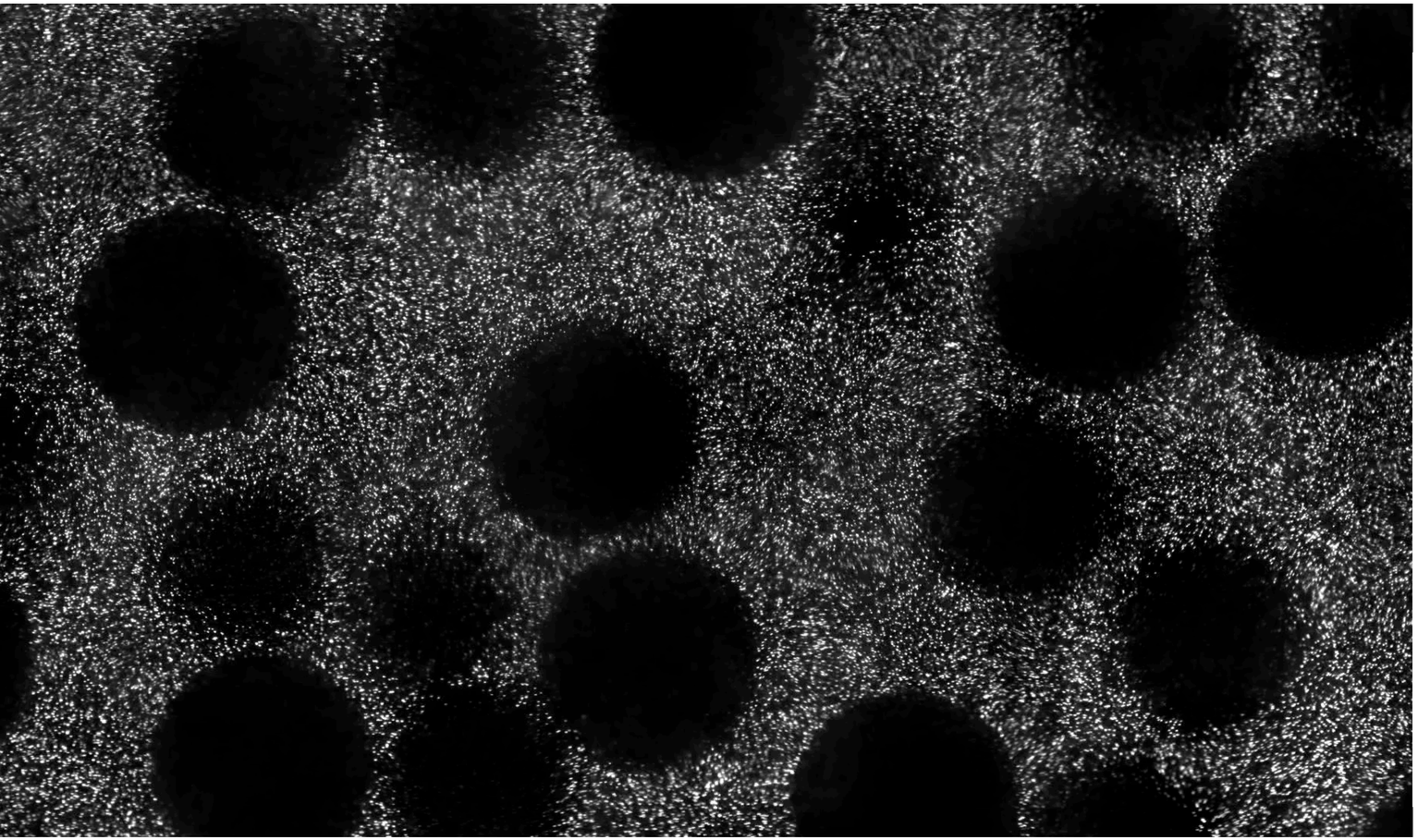
Imposed particle pressure

$$\eta_s = \frac{\mu(J(\phi))}{J}$$

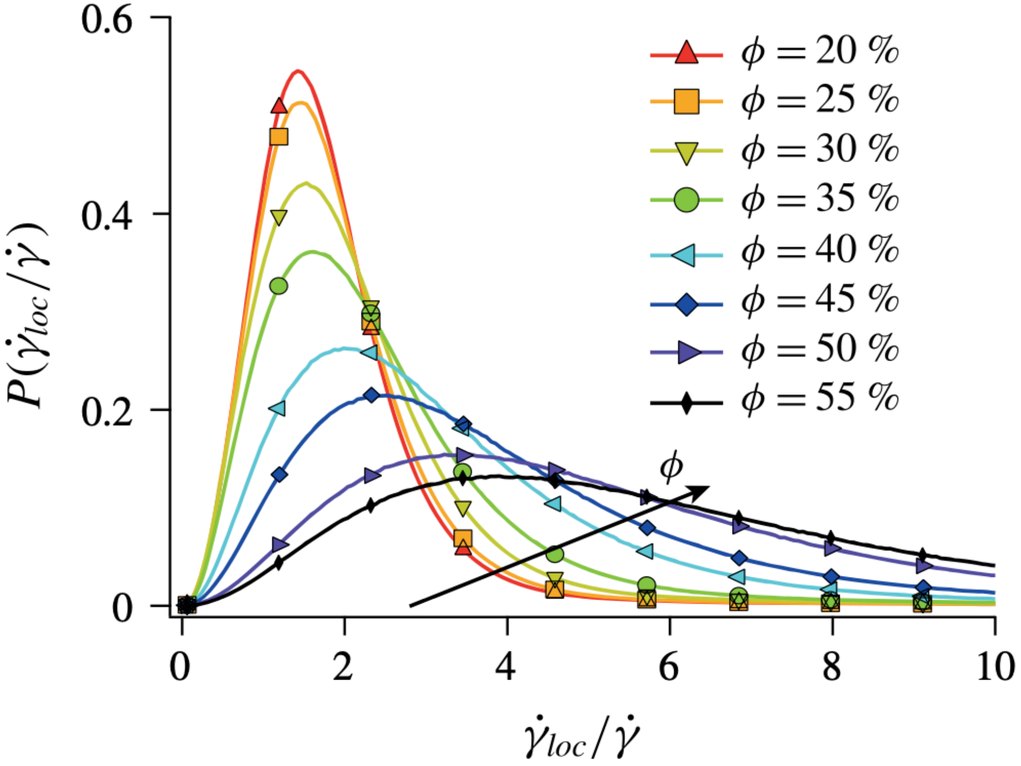


(Boyer, Guazzeli, Pouliquen PRL 2011)

Microscopic origin of the viscosity divergence



(Souzy JFM 2017)



$$\mathcal{P} \sim \eta_f \eta_s(\phi)\dot{\gamma}^2 \sim (1-\phi)\eta_f \dot{\gamma}_{\rm loc}^2 \longrightarrow \eta_s(\phi) \sim \frac{\dot{\gamma}_{\rm loc}^2}{\dot{\gamma}^2} = \mathcal{L}^2(\phi)$$

Power dissipation

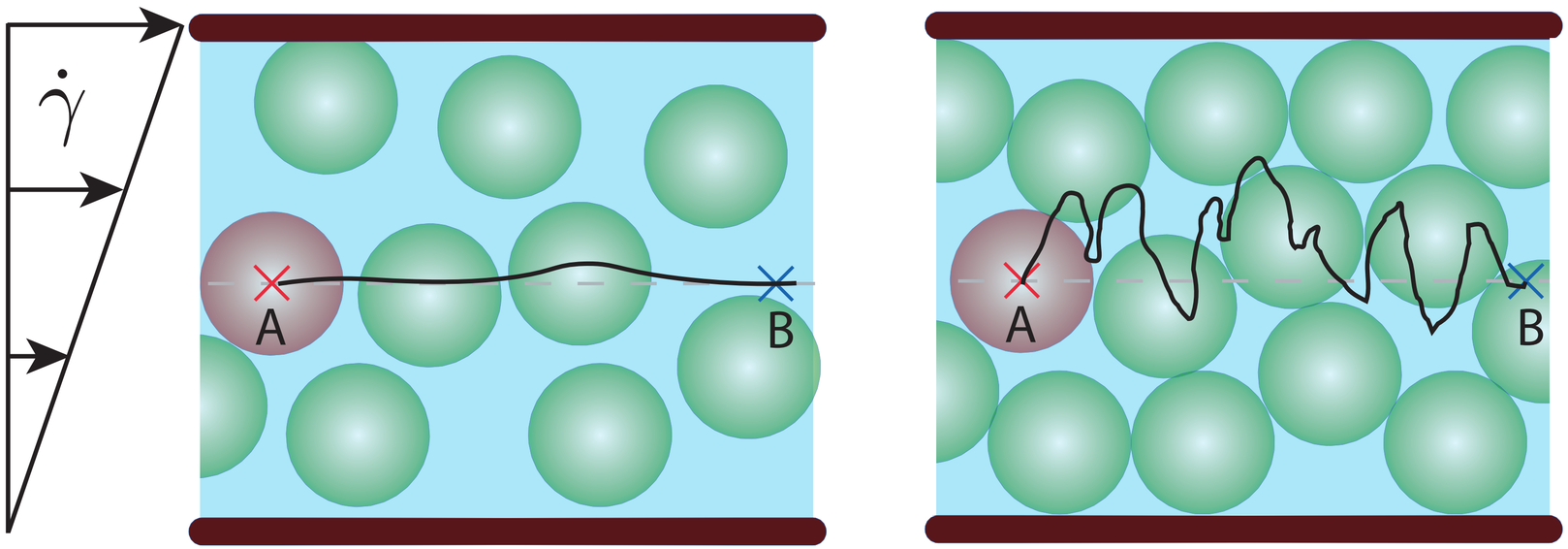
Lever effect

(DeGiuli, Lerner, Wyart PRE 2015)

Q: how velocity fluctuations are amplified as $\phi \to \phi_c$?

Floppy modes theory

(DeGiuli, Lerner, Wyart PRE 2015)



Geometry: particles cannot overlap!

$\downarrow \mathcal{L}(\phi)$

Increase of velocity fluctuations as $\phi \to \phi_c$

→

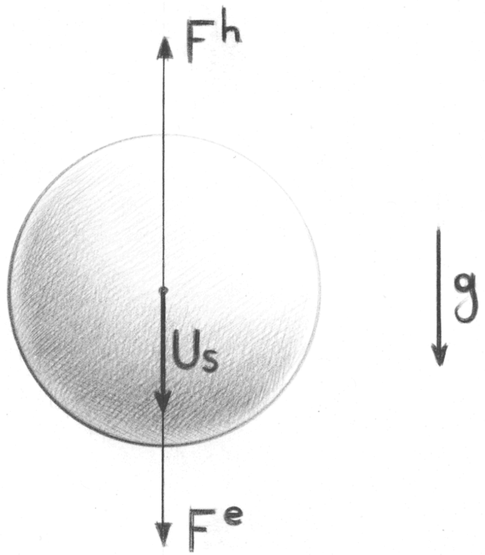
Scalings for frictionless spheres:

$$\eta_s(\phi) \sim \mathcal{L}^2(\phi) \sim (\phi_c - \phi)^{-2.8}$$

Scalings + DEM for frictional spheres:

$$\eta_s(\phi) \sim \mathcal{L}^2(\phi) \sim (\phi_c - \phi)^{-2}$$

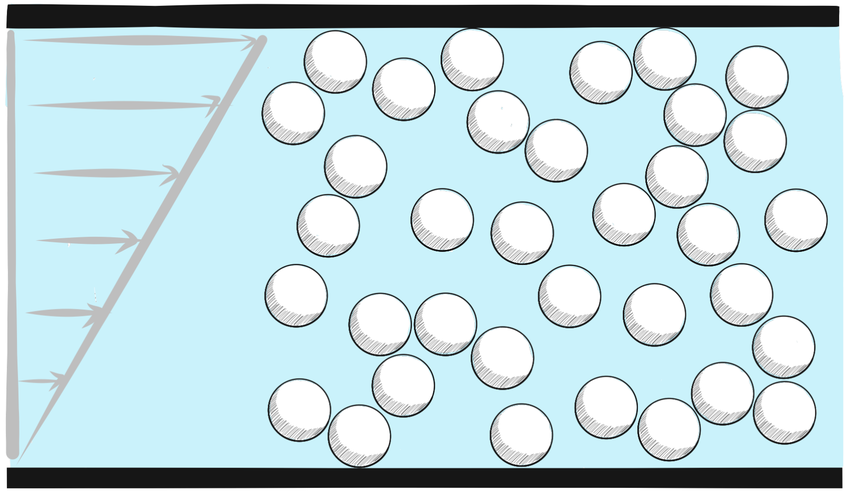
Sedimentation ($\rho_p > \rho_f$)



- Long range hydrodynamic interactions

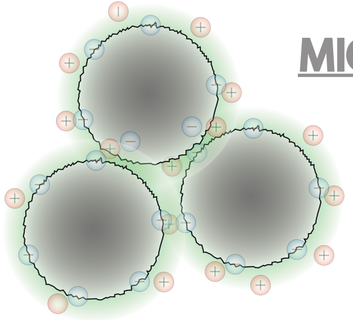
$\neq$

Rheology ($\rho_p = \rho_f$)



- Geometry (Solid contacts)

More recently…(on rheology)



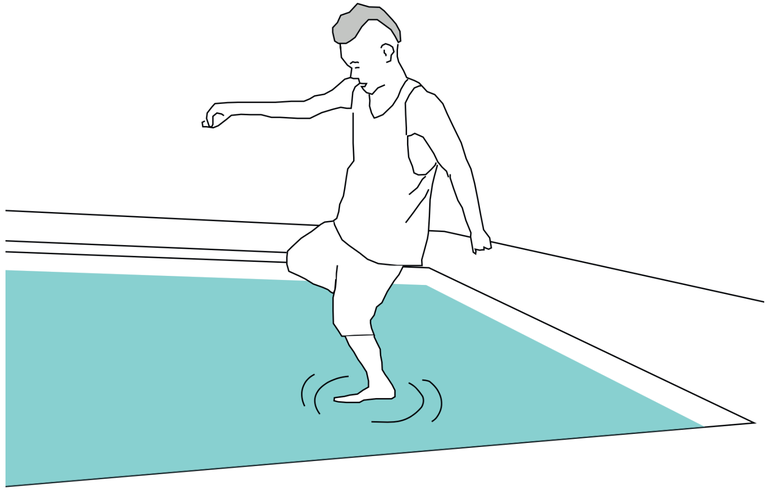
MICROSCOPIC

Particle friction

MACROSCOPIC

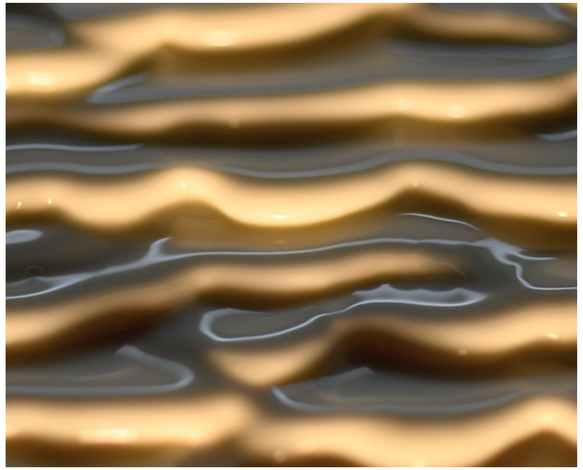
Rheological flow rules
Flowing properties

Shear thickening



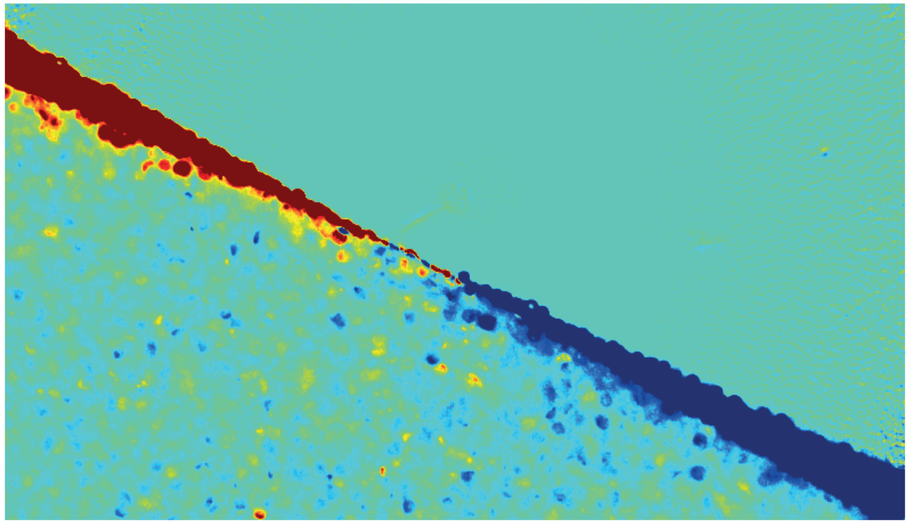
(Seto PRL 2013, Wyart & Cates PRL 2014
Clavaud PNAS 2017)

Flow instabilities



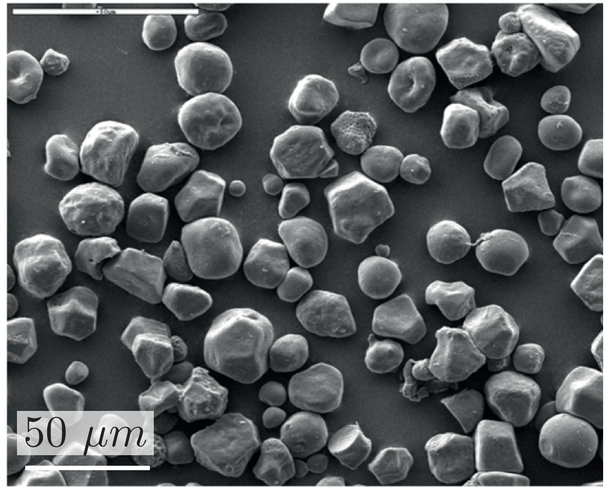
(Darbois CommPhys 2020)

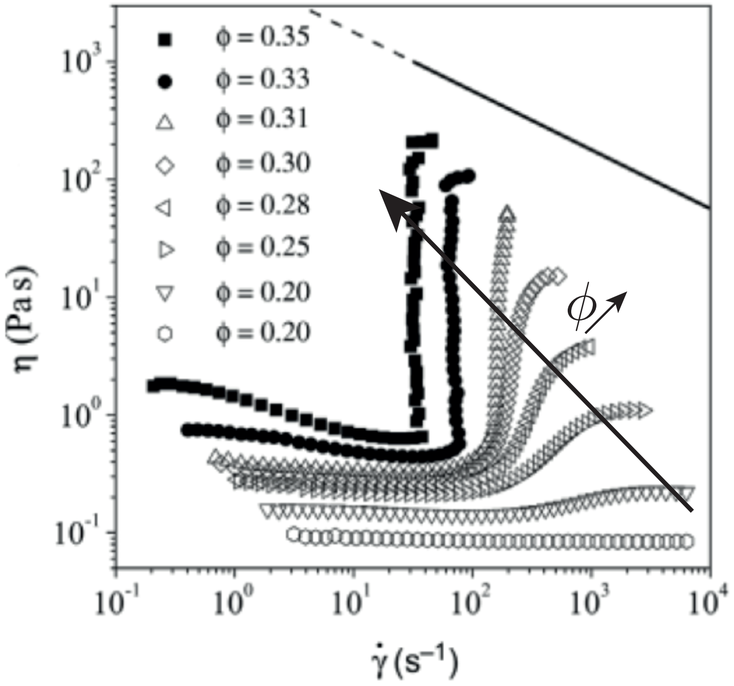
Hysteresis



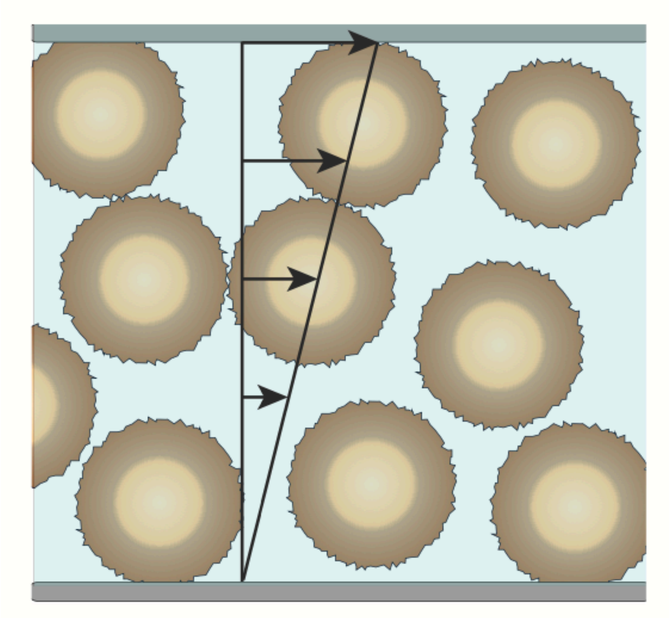
(Perrin PRX 2019)

What are shear thickening suspensions?



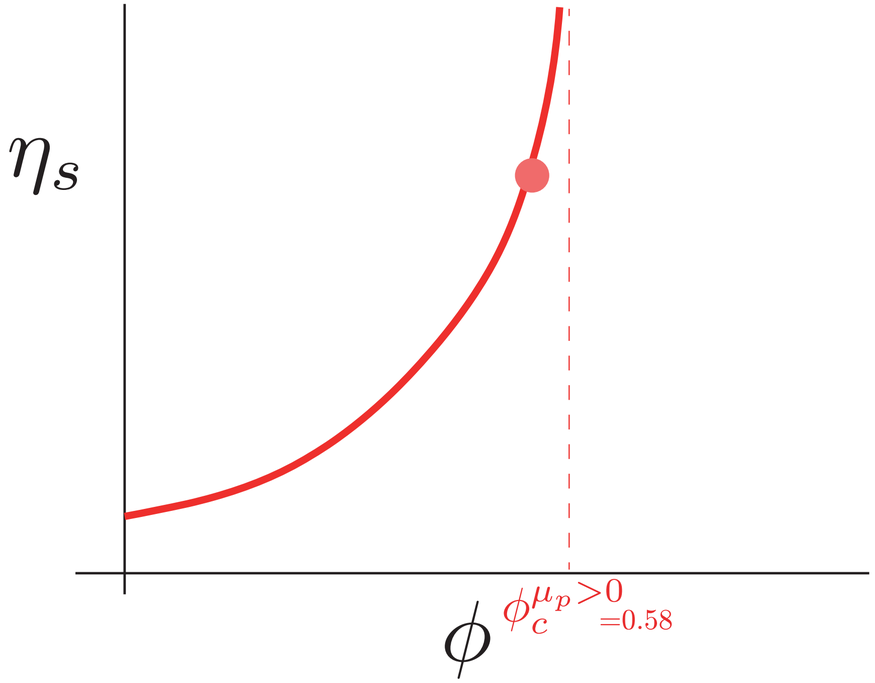


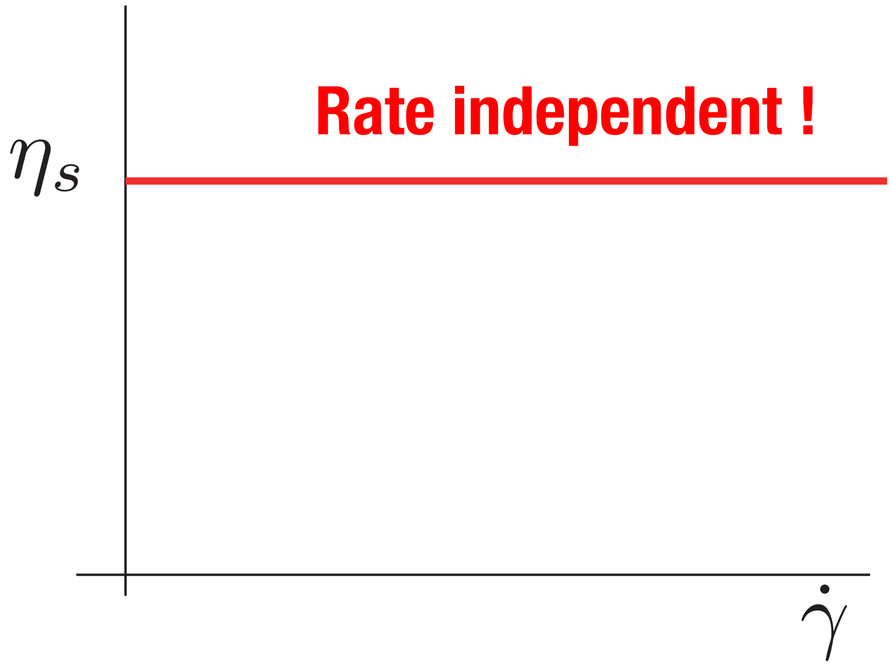
A puzzle ?



Granular Suspension

$$\rightarrow \frac{\eta_s}{\eta_f} = f(\phi) \sim (\phi_c - \phi)^{-2}$$

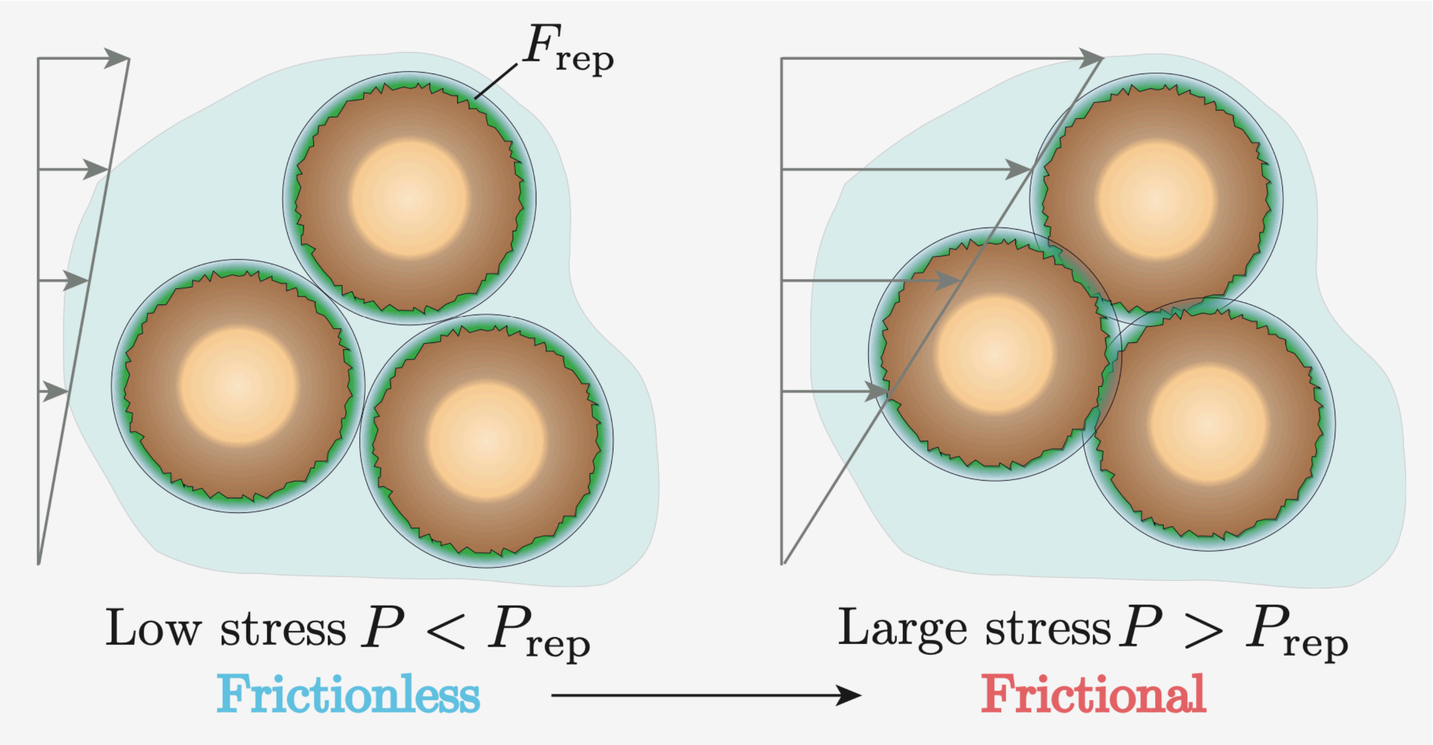




Origin of shear thickening

Key idea: introduce a short range repulsive force

$$F_{\text{rep}} \longrightarrow P_{\text{rep}} = \frac{F_{\text{rep}}}{d^2}$$

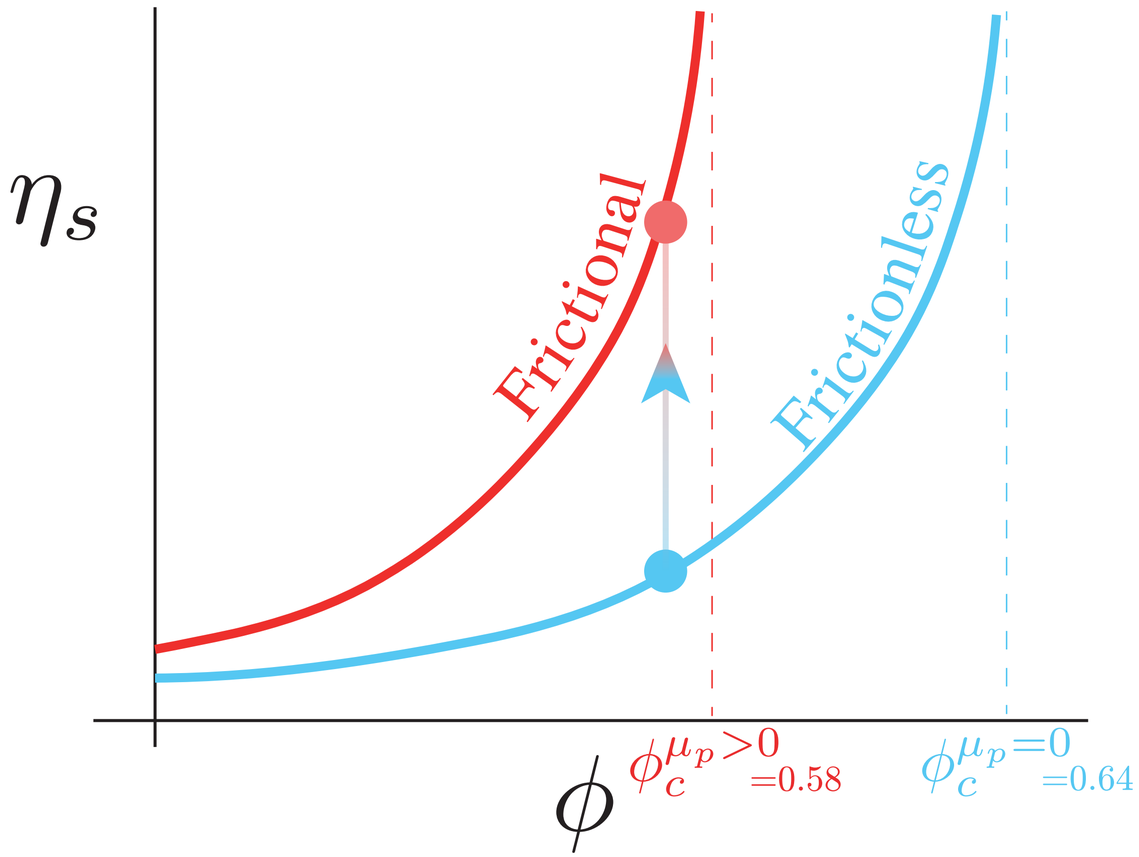


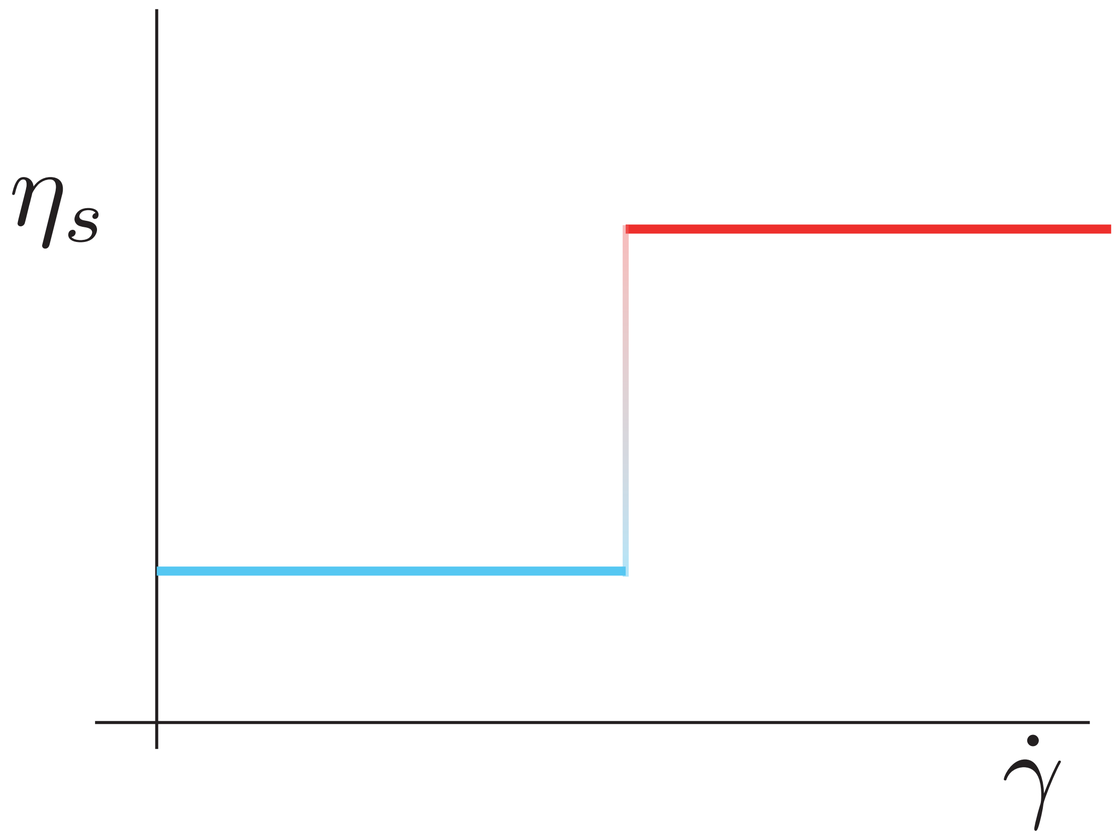
Shear thickening = frictional transition

(Seto PRL 2013, Mari JoR 2014)

Why so dramatic ?

ϕ_c depends on μ_p !





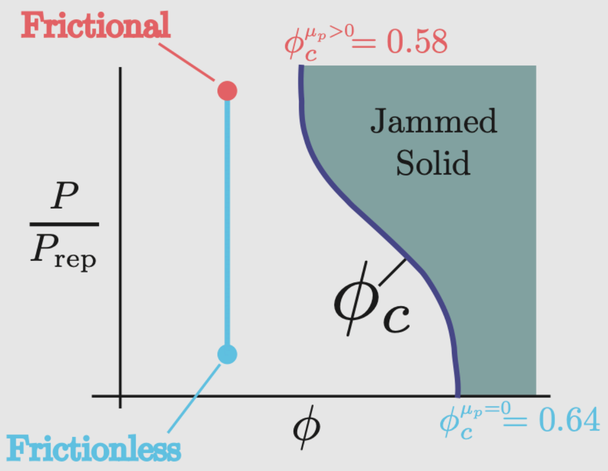
Frictional transition model: (Wyart & Cates PRL 2014)

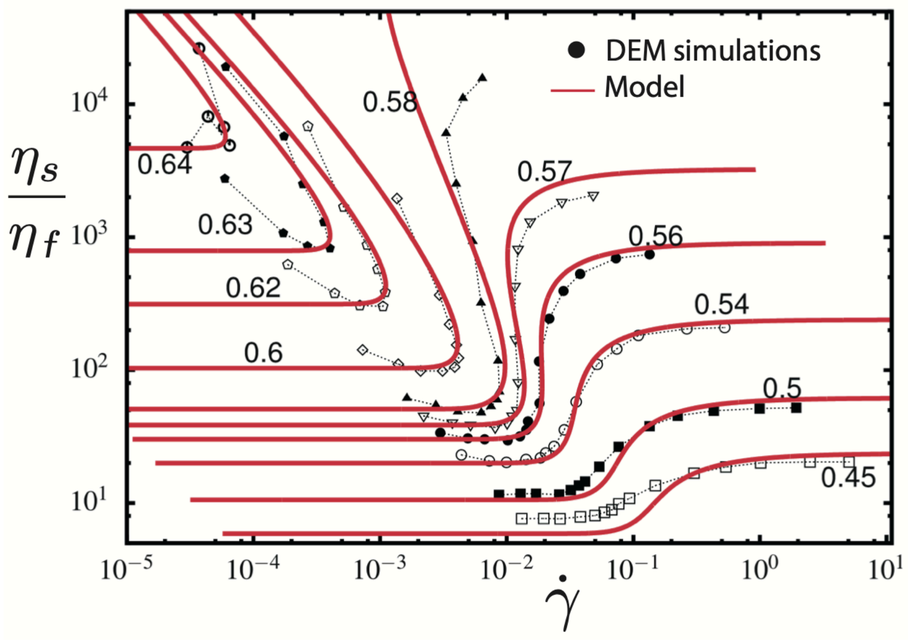
Order parameter: $f = 0 \rightarrow$ Frictionless when $P < P_{\text{rep}}$

$f = 1 \rightarrow$ Frictional when $P > P_{\text{rep}}$

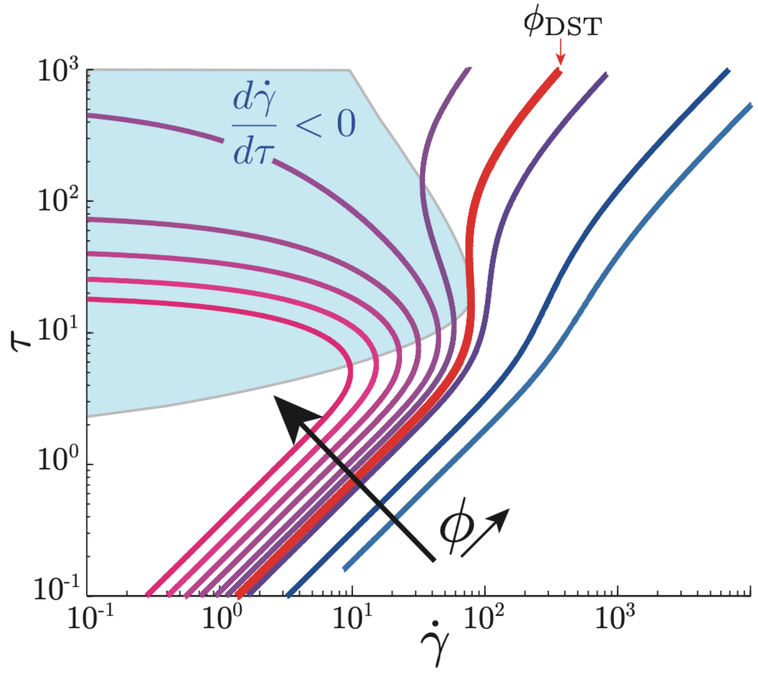
$$\phi_c\left(\frac{P}{P_{\text{rep}}}\right) = \phi_c^{\mu_p=0}(1-f) + \phi_c^{\mu_p=1} f$$

$$\frac{\eta_s}{\eta_f} = \left(\phi_c\left(\frac{P}{P_{\text{rep}}}\right) - \phi\right)^{-2}$$





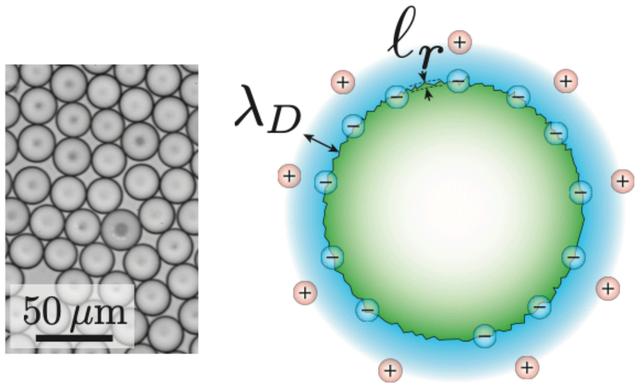
(Singh JoR 2018)



Activation of friction $\rightarrow$ negatively sloped rheograms !

Experimental validation

Silica beads 25 microns+water+NaCl

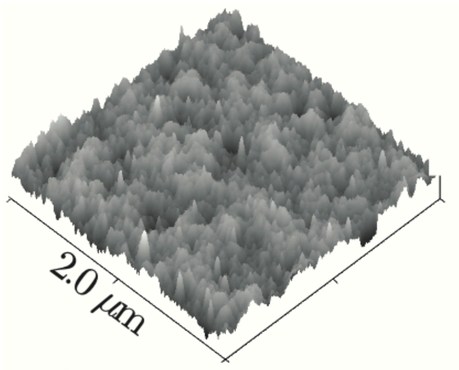


Repulsive force:

$$F_{\text{rep}} = F_0 \exp(-r/\lambda_D)$$

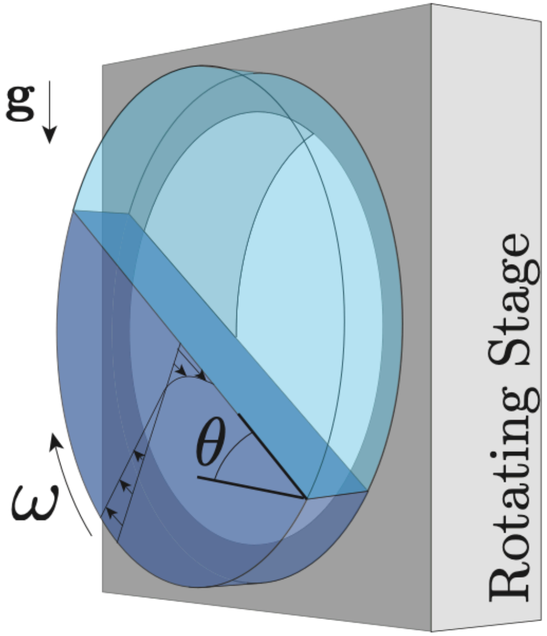
Debye length:

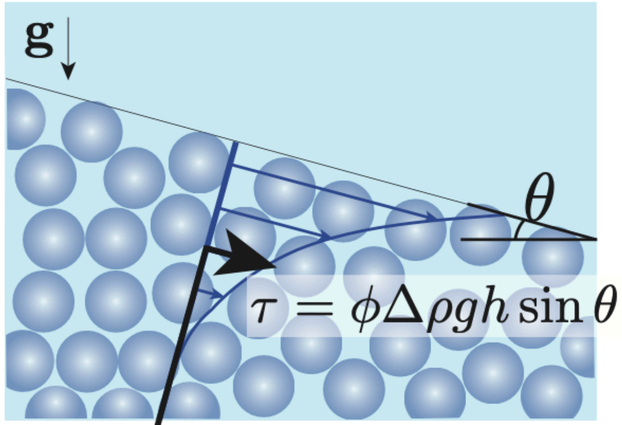
$$\lambda_d = \frac{0.304}{\sqrt{[NaCl]}} \text{ (nm)}$$



$\ell_r = 3.73 \pm 0.80$ nm

Access to friction: pressure imposed



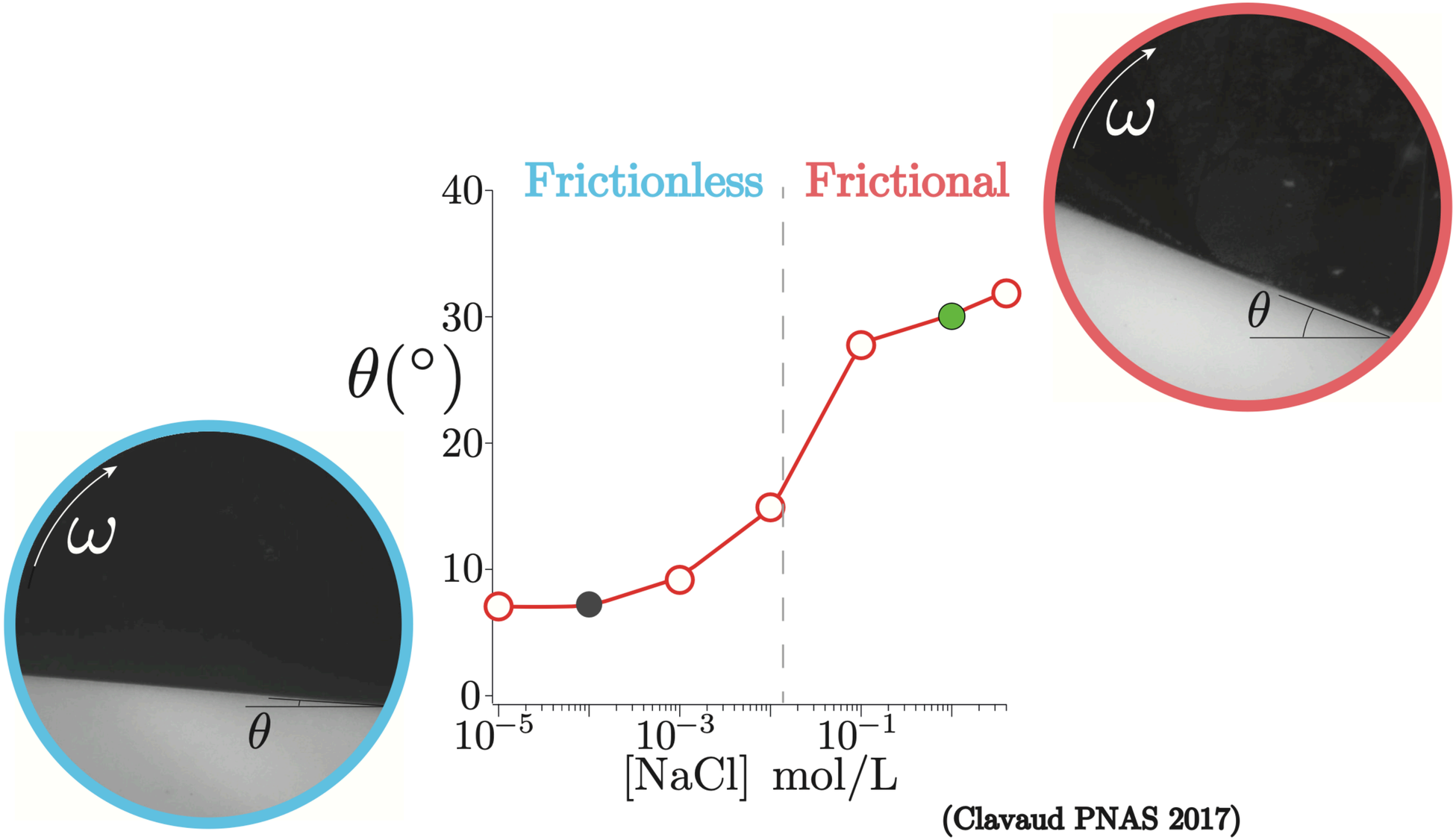


$\tau = \phi \Delta\rho g h \sin\theta$

$P = \phi \Delta\rho g h \cos\theta \sim 1$ Pa

$$\mu_c = \frac{\tau}{P} = \tan\theta$$

(Clavaud PNAS 2017)



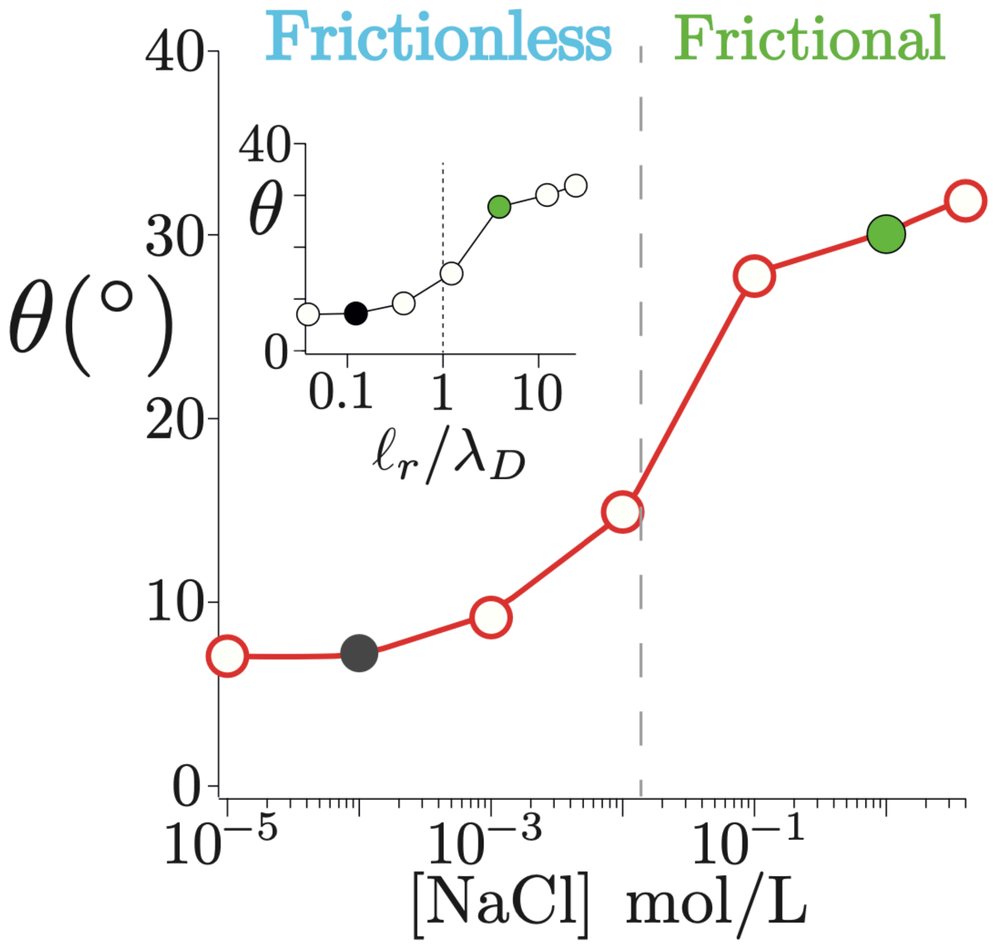
(Clavaud PNAS 2017)

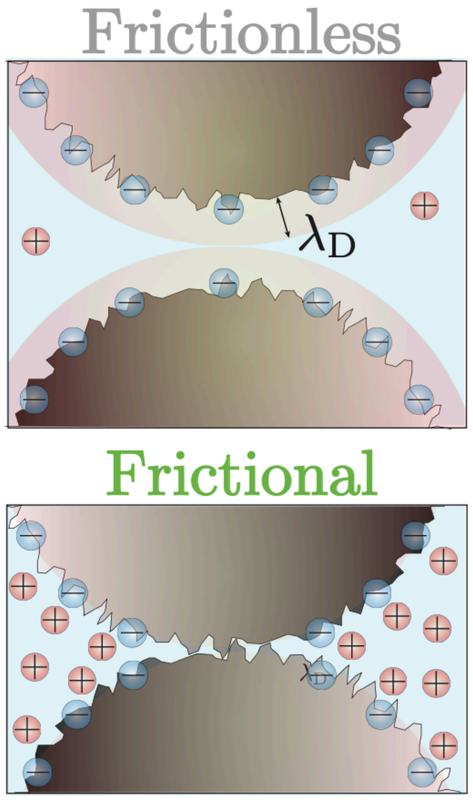
Repulsive force → $\theta = 6°$ → **frictionless grains !** (Peynaud & Roux PRE 2008)

~~**Repulsive force**~~ → $\theta \approx 30°$ → **frictional grains !**

(Clavaud PNAS 2017)

Experimental validation



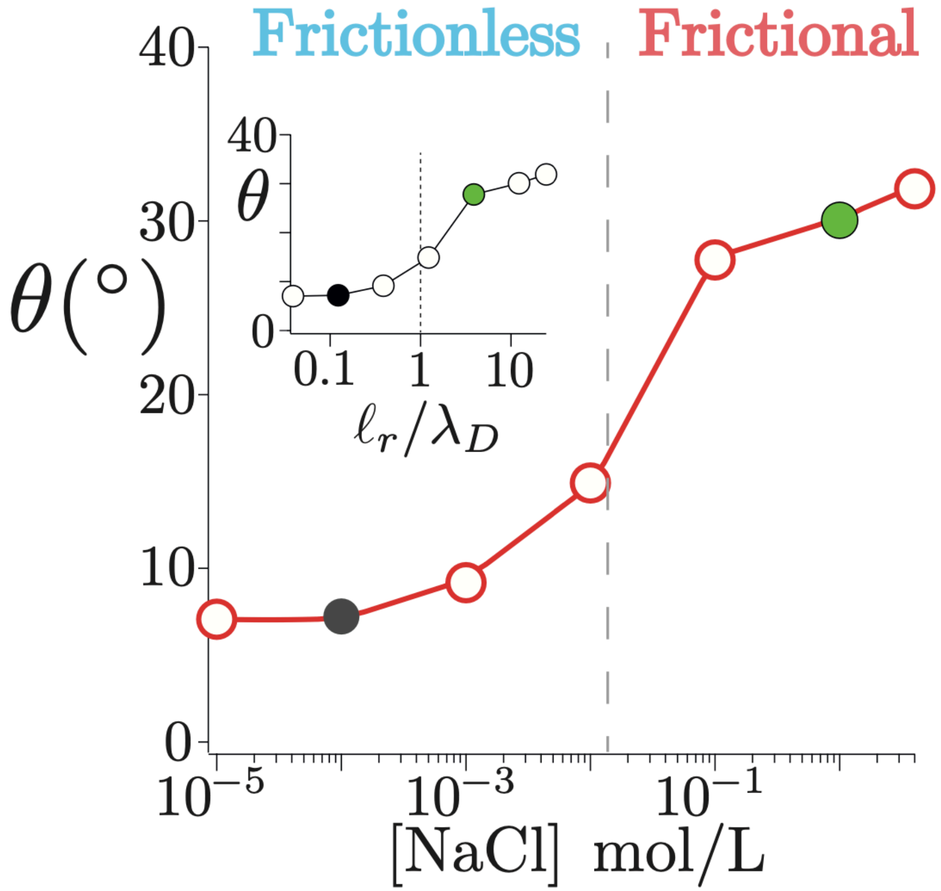


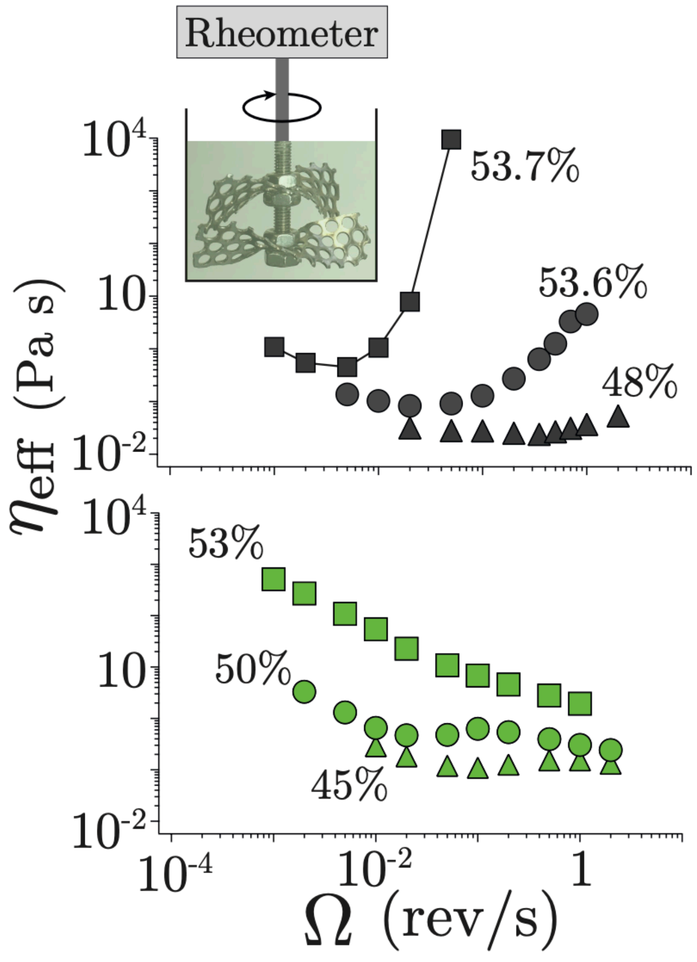
(Clavaud PNAS 2017)

Nano-metric features control the macroscopic response of the system!

(Clavaud PNAS 2017)

Experimental validation





Repulsive force

~~Repulsive force~~

Repulsive force → Frictionless state under low stress → Shear thickening

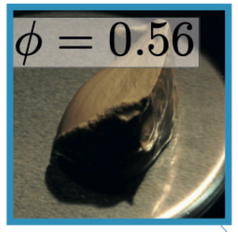
Validation Frictional Transition Model

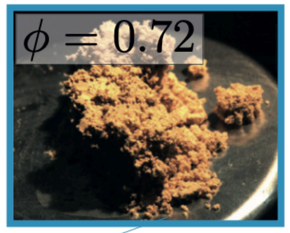
(Clavaud PNAS 2017)

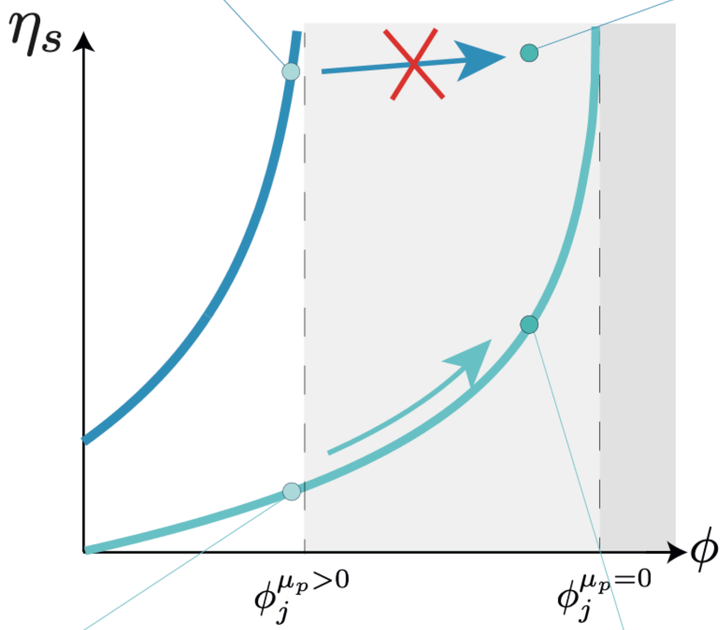
Microscopic friction → Macroscopic rheology

- Optimize modern concretes

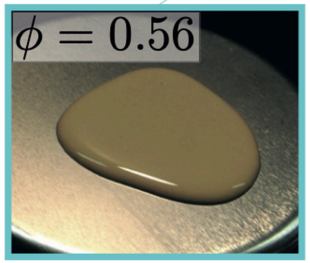
CaCO3+H20

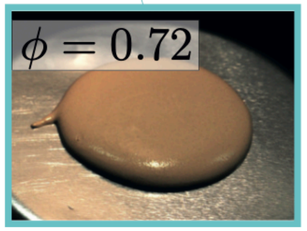


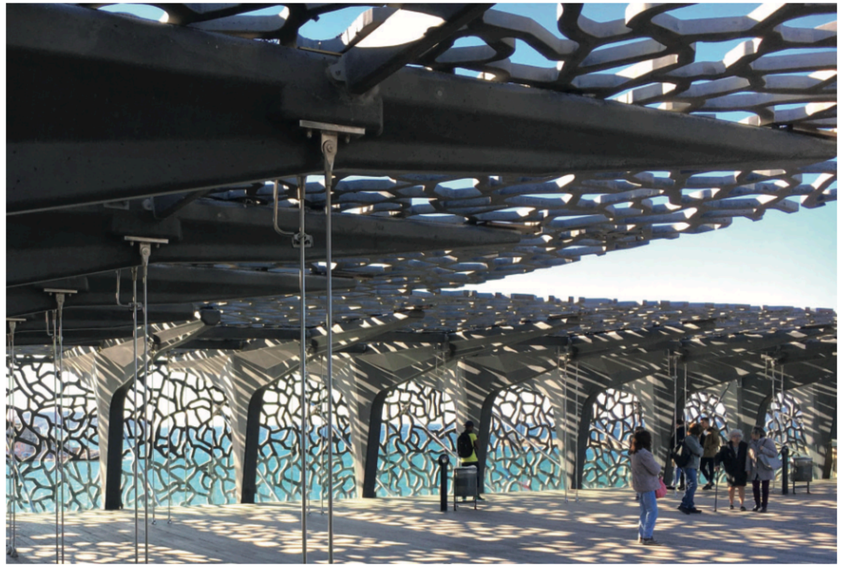




CaCO3+H20+Polymer







Collaboration: Chryso
ANR: ScienceFriction

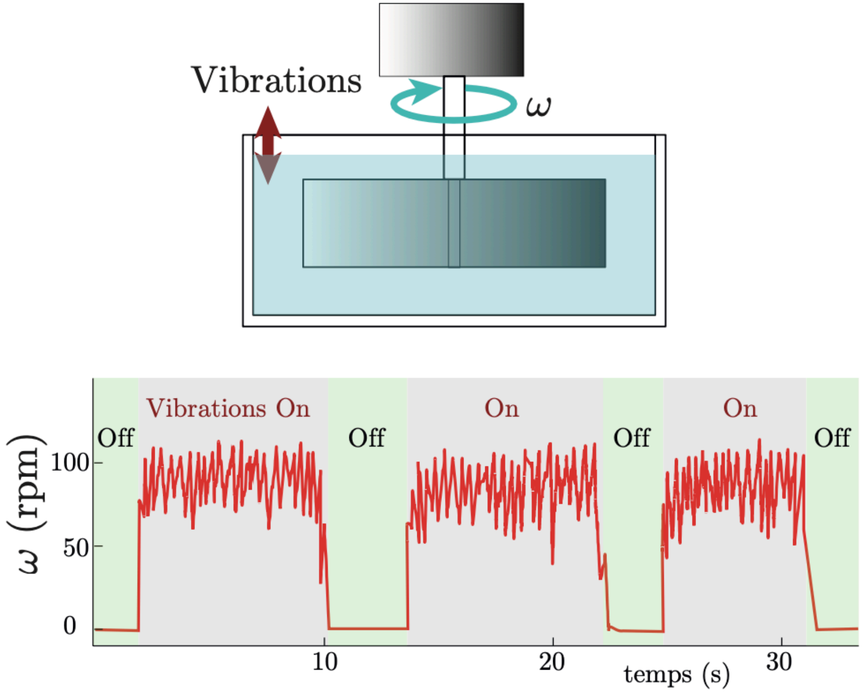
(Richards Rheol. Acta 2020)

Microscopic friction → Macroscopic rheology

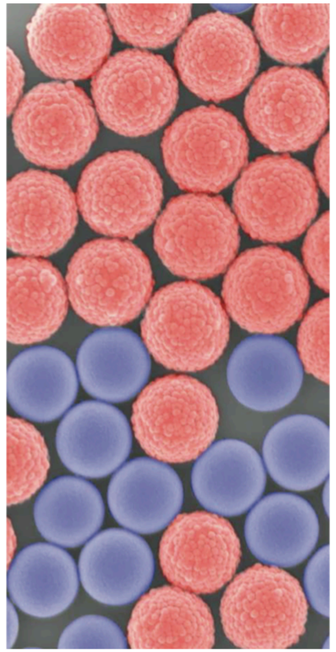
- Control of the flow properties by controlling friction ?

Microscopic friction → Macroscopic rheology

- Control of the flow properties by controlling friction ?



(Lin PNAS 2016)



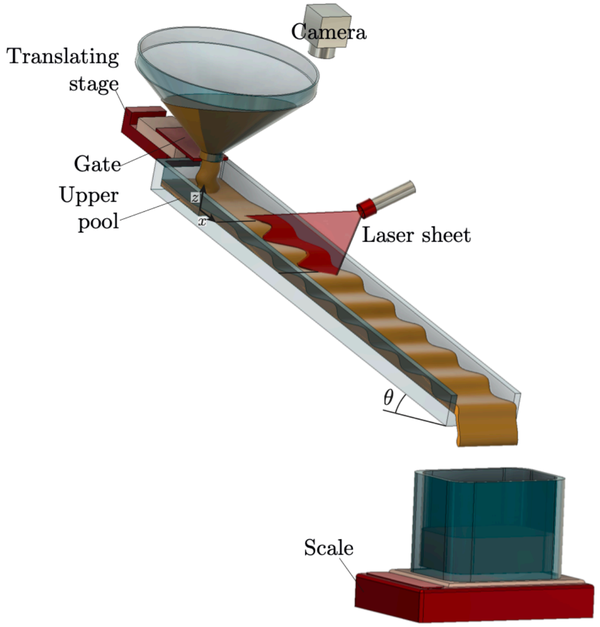
(Isa PNAS 2018)

Microscopic friction → Macroscopic rheology

- Go beyond rheology: hydrodynamics

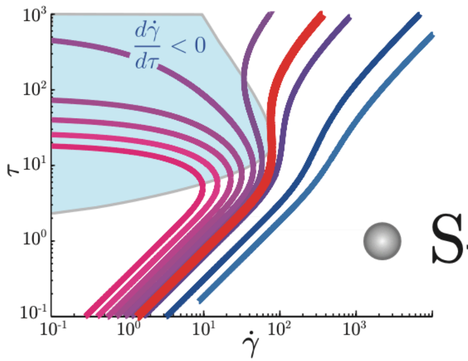
Shear thickening suspension down an incline

B. Darbois Texier H. Lhuissier Y. Forterre



Not understood, no flow rules !

(Balmforth Phys. Let. A 2005)



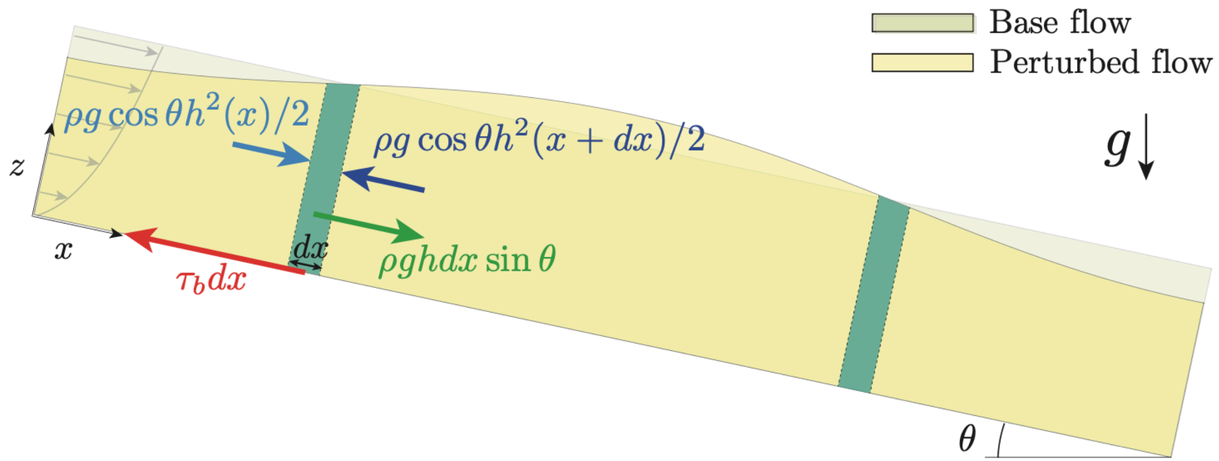
- S-shaped $A = \dfrac{d\dot{\gamma}}{d\tau} < 0$
- No inertia $Re \ll 1$

Force balance (depth averaged):

$$\tau_b = \rho g h \sin\theta$$

Basal stress Gravity

- S-shaped $A = \dfrac{d\dot{\gamma}}{d\tau} < 0$
- No inertia $Re \ll 1$

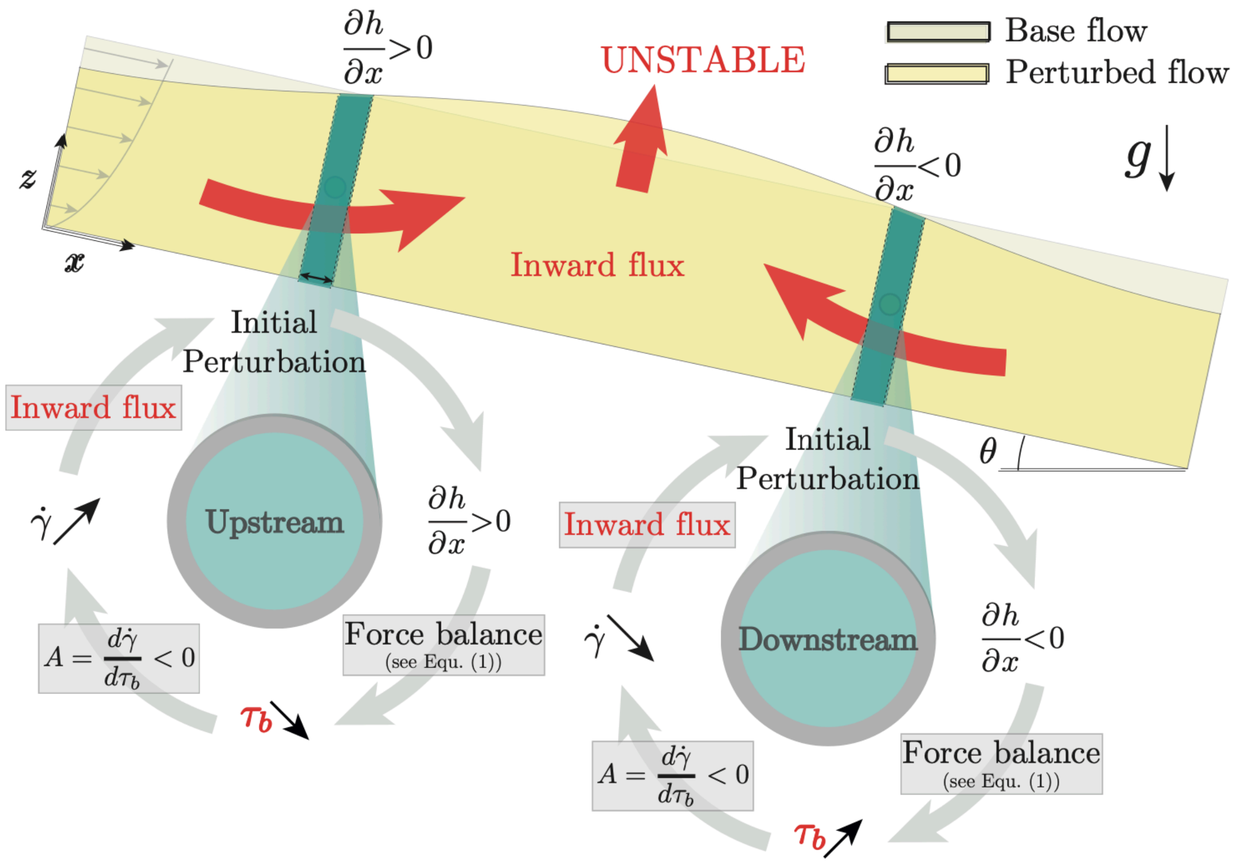


Force balance (depth averaged):

$$\tau_b = \rho g h \sin\theta - \rho g h \cos\theta \frac{\partial h}{\partial x}$$

Basal stress Gravity Hydrostatic pressure

- S-shaped $A = \dfrac{d\dot{\gamma}}{d\tau} < 0$
- No inertia $Re \ll 1$



Force balance (depth averaged):

$$\tau_b = \rho g h \sin\theta - \rho g h \cos\theta \frac{\partial h}{\partial x}$$

Basal stress Gravity Hydrostatic pressure

$\rightarrow$ Unstable if $A = \dfrac{d\dot{\gamma}}{d\tau} < 0$

Linear stability analysis (depth averaged)

(Mass conservation + Force balance without inertia)

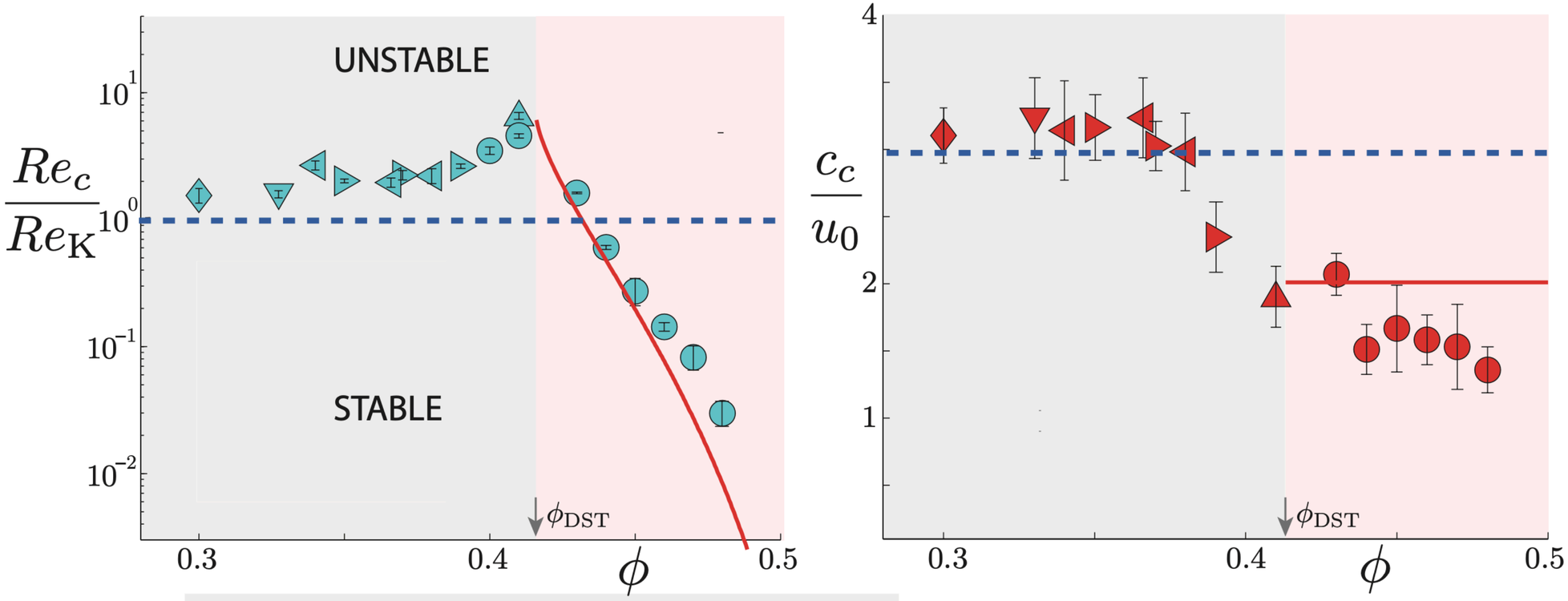
Layer thickeness perturbation:

$$\boxed{\frac{\partial h_1}{\partial t} + \tilde{c}\frac{\partial h_1}{\partial x} = \frac{A}{\tan\theta}\frac{\partial^2 h_1}{\partial x^2}} \quad \text{with} \quad \tilde{c} = 2 + A$$

→ Newtonian fluid $\quad A = \frac{d\dot{\gamma}}{d\tau} > 0 \quad$ Stable

→ Shear thickening (S-shaped) $\quad A = \frac{d\dot{\gamma}}{d\tau} < 0 \quad$ Unstable

The bump ``anti-diffuses'' !



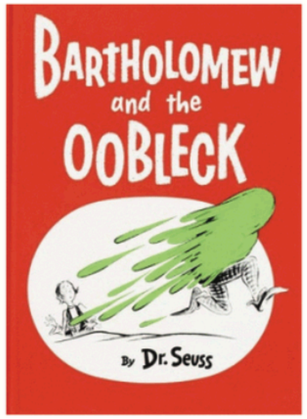
Kapitza (Roll waves) instability

Inertia $$Re_K = \frac{5}{6\tan\theta}$$

New instability

Coupling negatively sloped rheology/free surface → Oobleck waves

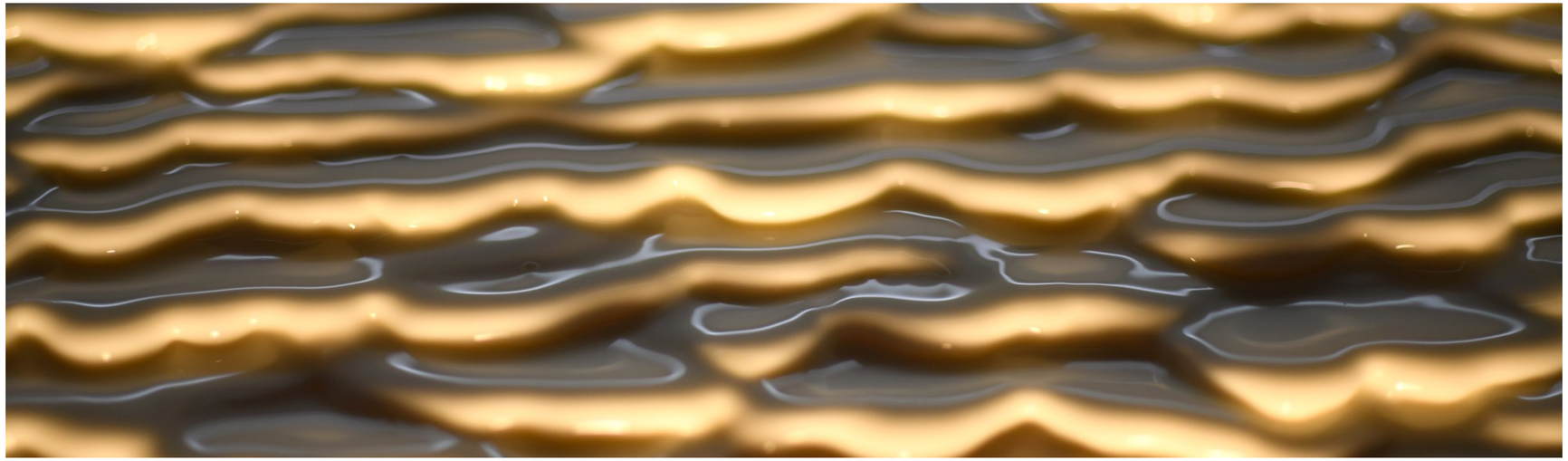
$$Re_c = \frac{3\tau_c{}^3\left(\phi_c - \phi\right)^4}{4\eta_s^2\rho g^2\sin^2\theta} \qquad c_c/u_0 = 2$$



(Dr. Seuss 1949)

(Darbois CommPhys 2020)

Shear thickening suspension down an incline

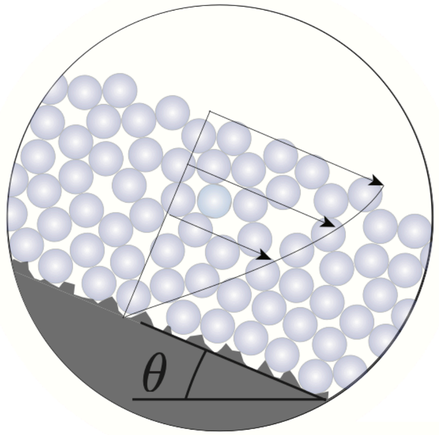


Frictional transition → S-shaped flow curve + Coupling with free surface deformation → Kinematic waves **unstable without inertia**

New class of unstable waves: extend to velocity-weakening materials (granular, micelles, mud…flows)?

Non-linear description?

Hysteresis of the avalanche angle



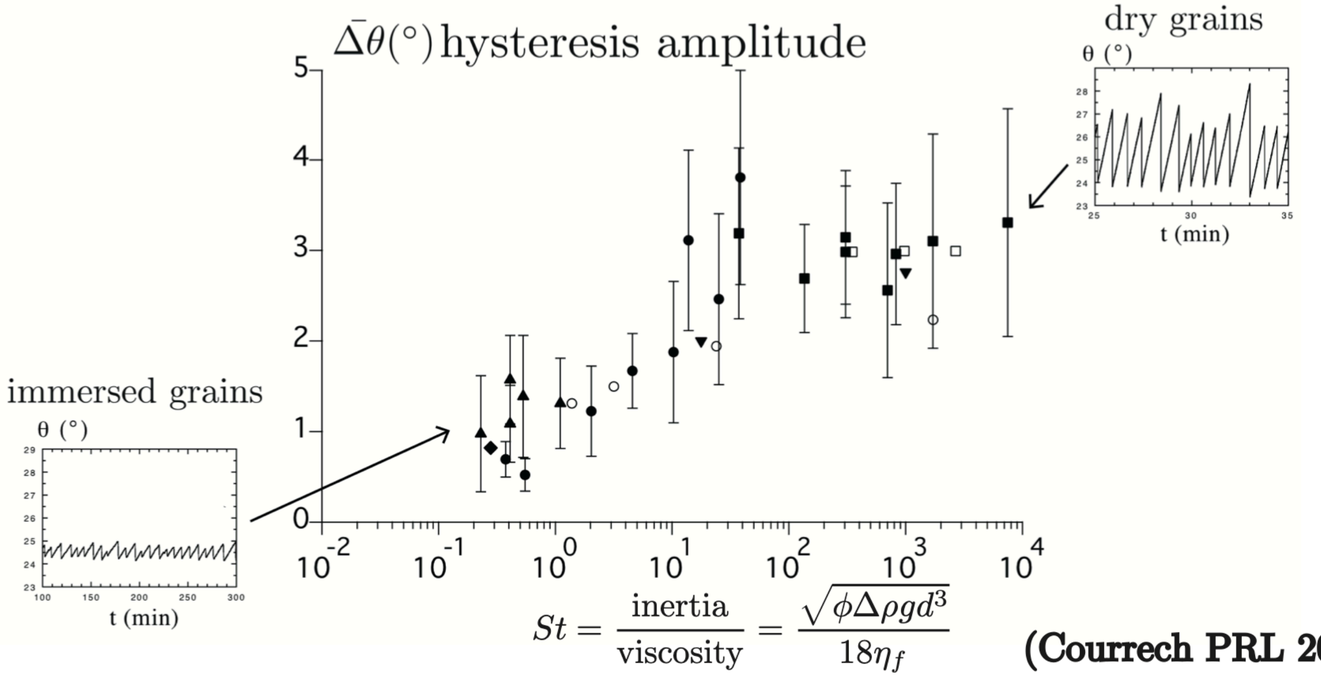
Hysteresis ⟶ Lanslides

$$\Delta\theta = \theta_{\text{start}} - \theta_{\text{stop}}$$

Tac-Tac model
(Quartier PRE 2000)

Self-induced fluidization
(DeGiuli PNAS 2017)

Models based on inertia



(Courrech PRL 2013)

Hysteresis should vanish without inertia

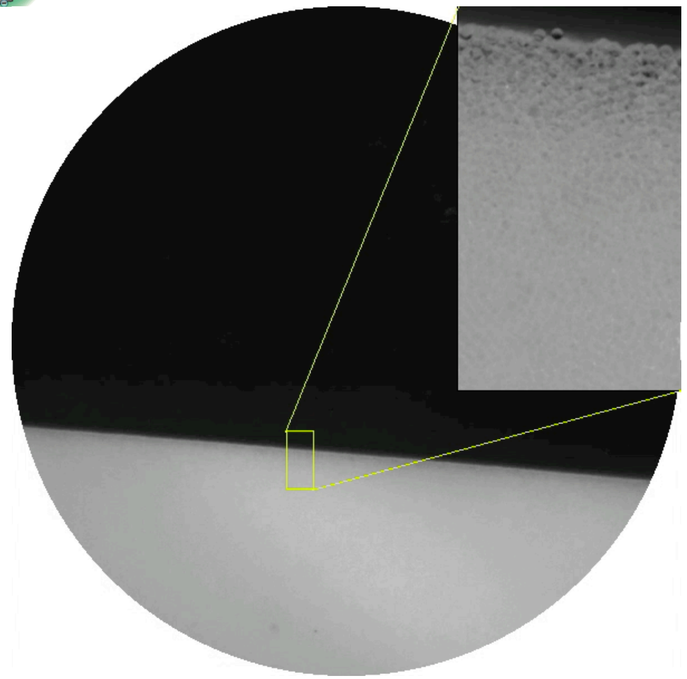
Hysteresis of the avalanche angle

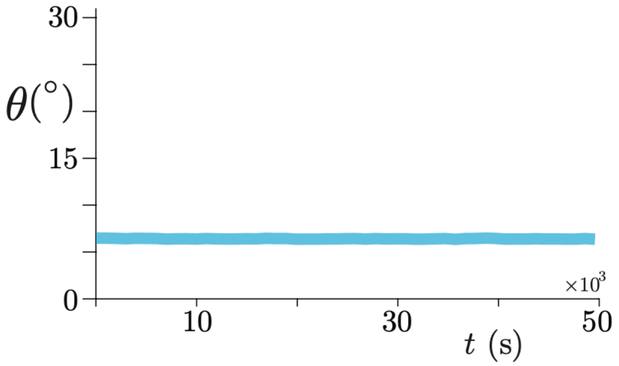
H. Perrin

Y. Forterre

M. Wyart

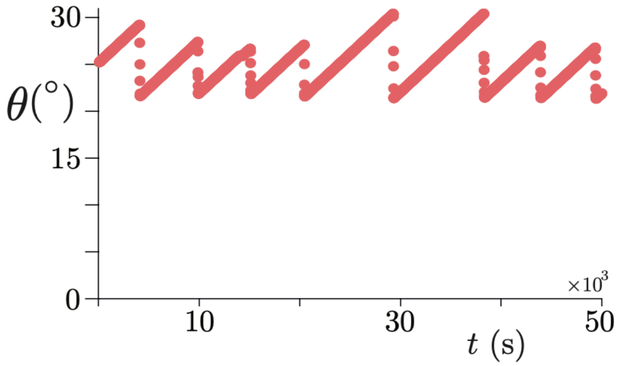
Frictionless particles



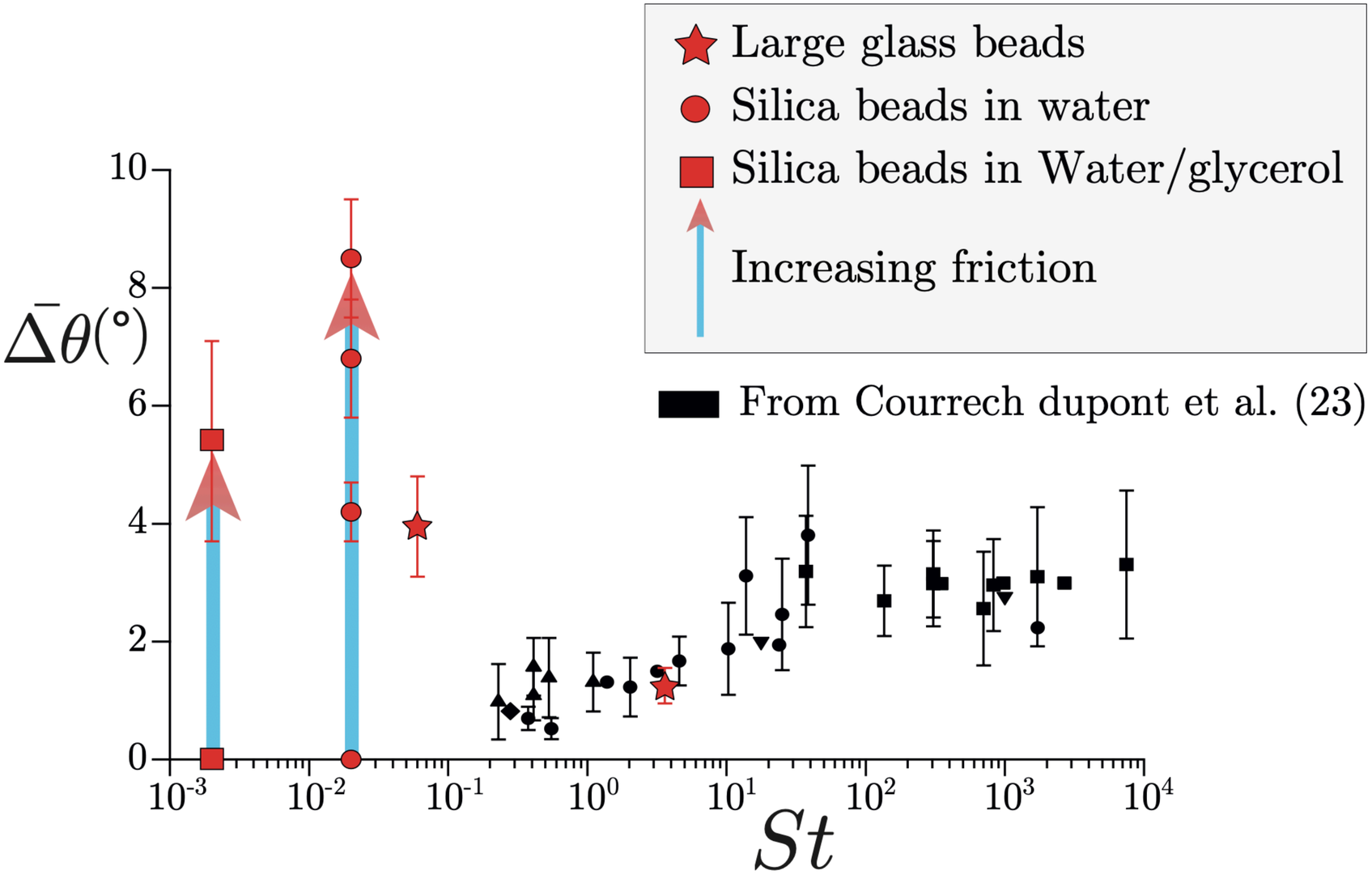


No friction⟶No hysteresis

Frictional particles



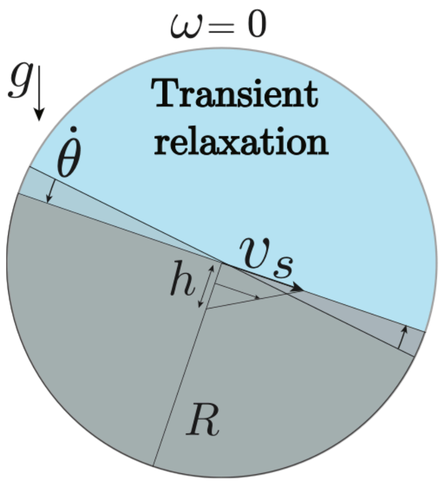
Hysteresis without inertia (St<<1) !

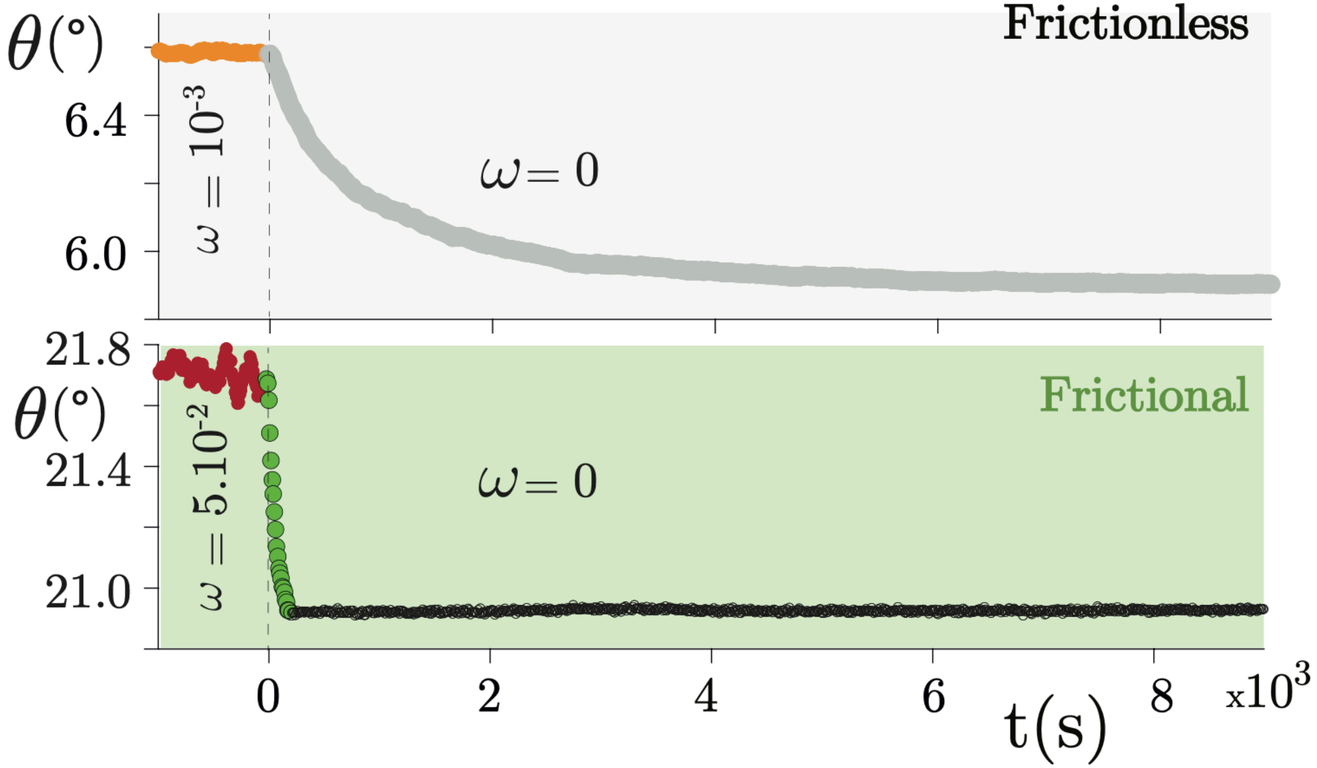


Key ingredient for hysteresis: friction not inertia !

(Perrin PRX 2019)

Link with rheology ?

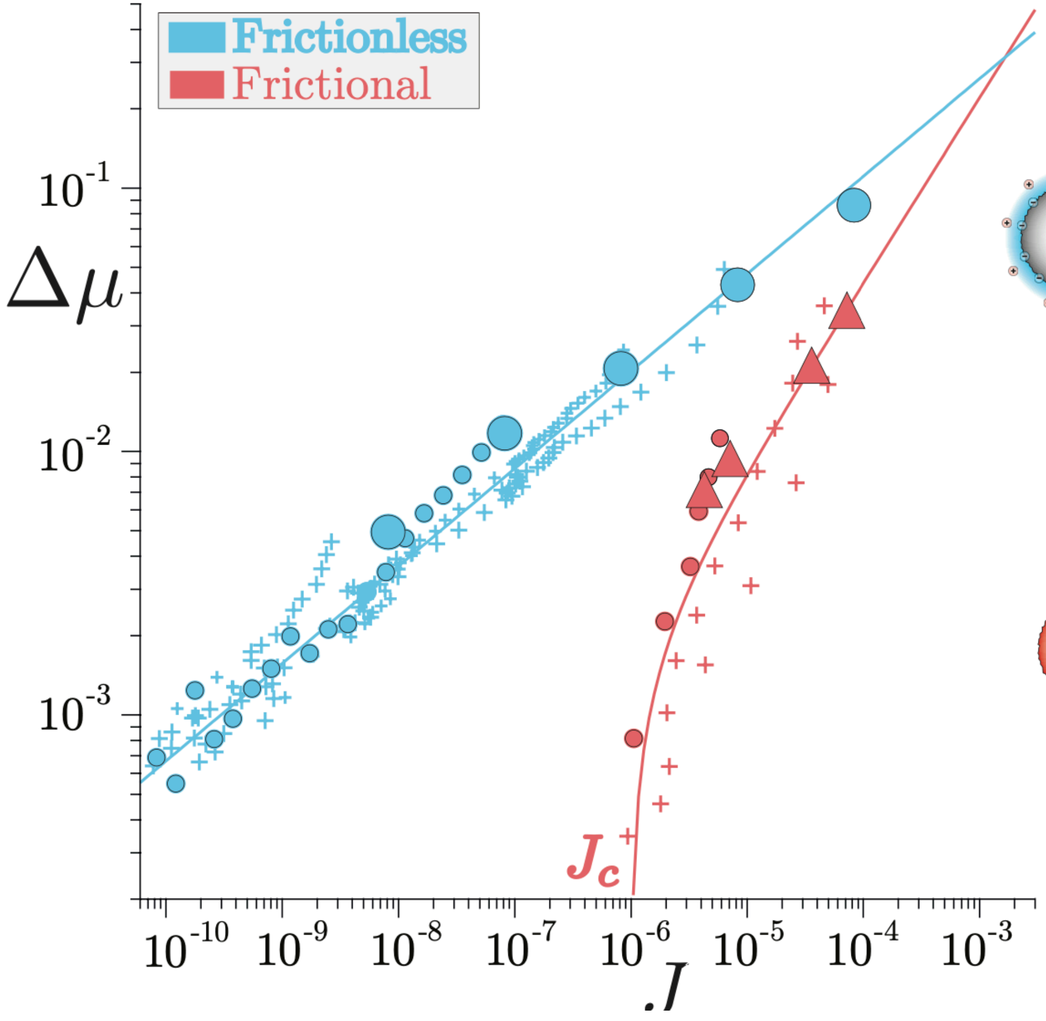


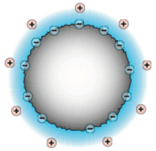


$$\mu = \tan\theta$$

$$J = \frac{\eta_f \dot{\gamma}}{P} = \frac{\eta_f}{P}\frac{\dot{\theta} R^2}{h^2}$$

Link with rheology:





Frictionless: monotonic rheology

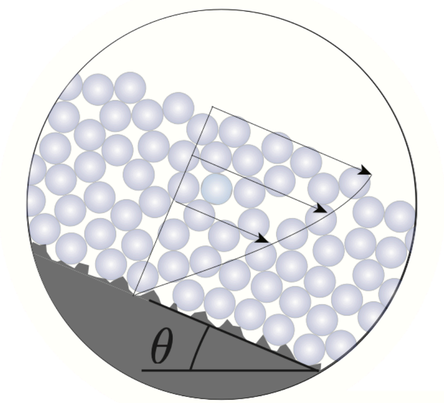
$$\Delta\mu = \mu - \mu_c^0 \sim J^{0.37}$$

consistent with predictions of DeGiuli PRE 2015.

Frictional: non-monotonic rheology

$$\Delta\mu = \mu - \mu_c^1 \sim (J - J_C)^{0.7}$$

Hysteresis of the avalanche angle

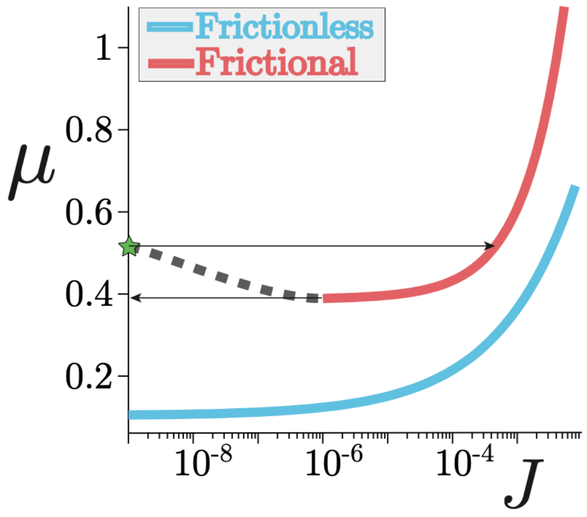


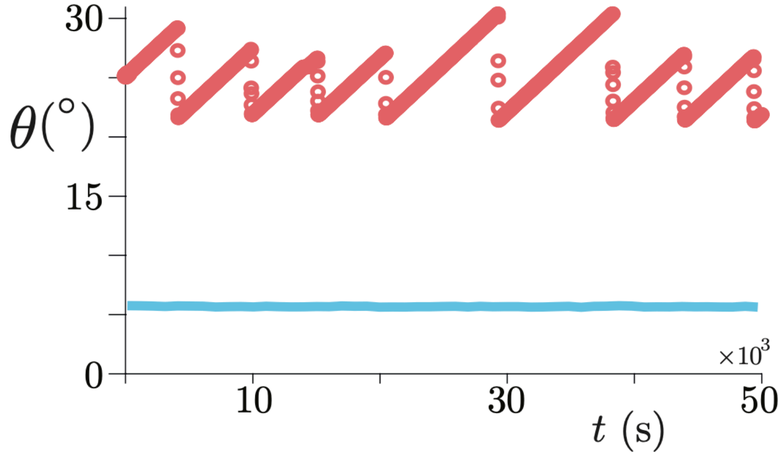
Hysteresis

$$\Delta\theta = \theta_{\text{start}} - \theta_{\text{stop}}$$

Inertia not required

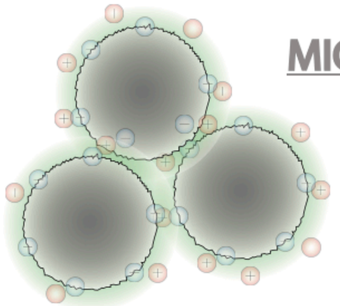
Inter-particle friction → Rheology → Flowing property





(Perrin PRX 2019)

Origin of velocity weakening?



MICROSCOPIC

Particle friction

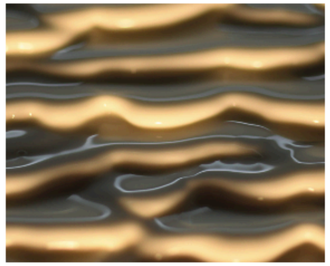
MACROSCOPIC

Rheological flow rules
Flowing properties

Shear thickening

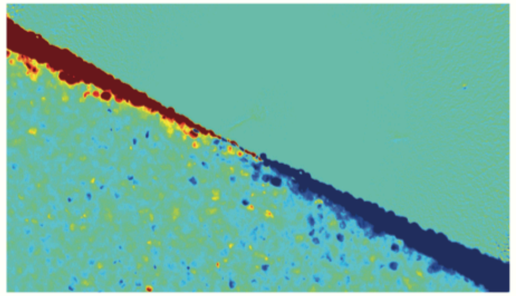
Frictional transition

Flow instabilities



Coupling
S-shaped rheology/free surface

Hysteresis



Hysteresis without inertia
non-monotonic flow rules

C. Clavaud

A. Bérut

H. Perrin

B. Darbois Texier

H. Lhuissier

Y. Forterre

M. Wyart

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European Research Council
Established by the European Commission

PlantMove

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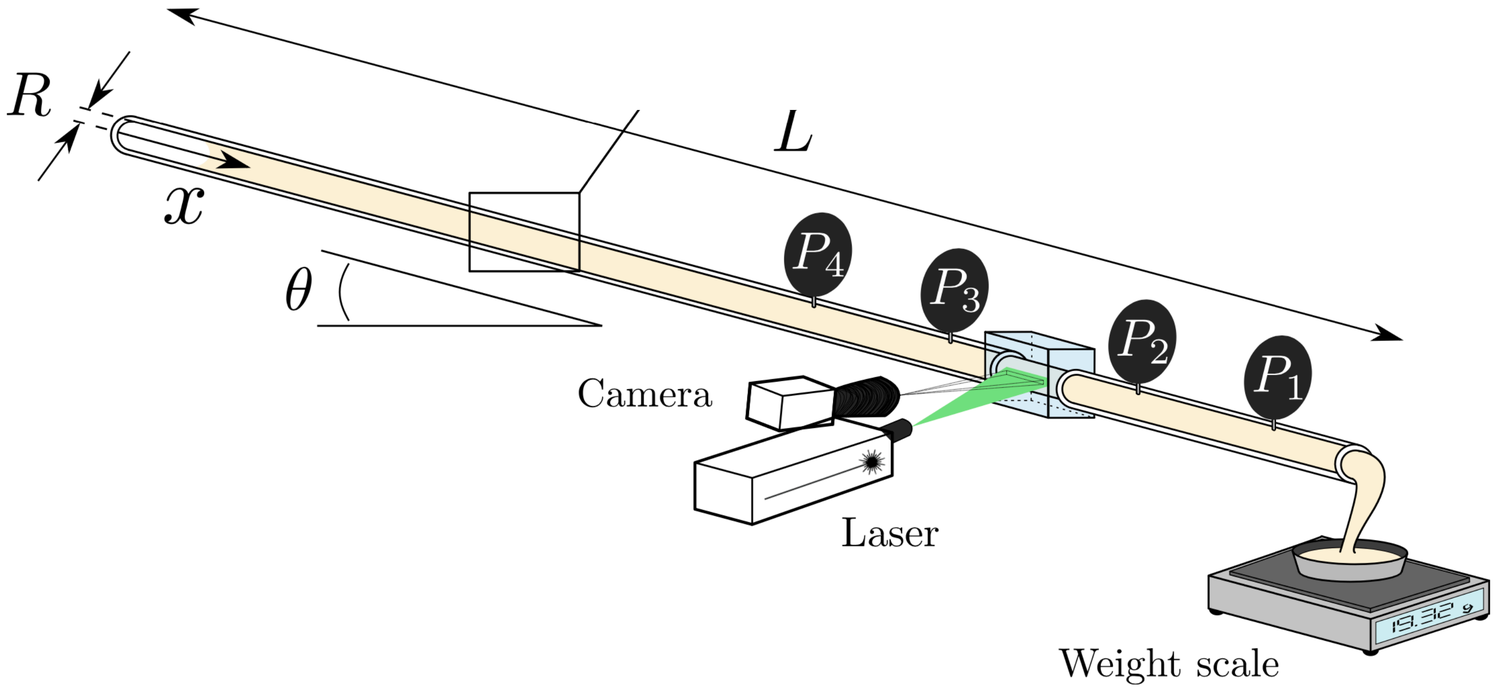
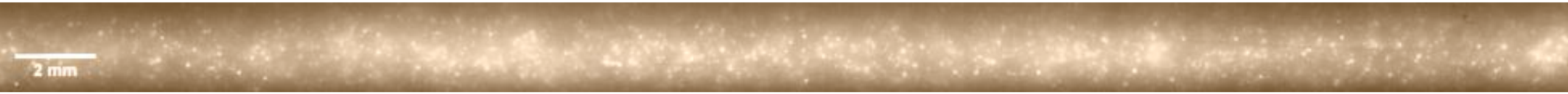
ANR
ScienceFriction

Pipe Flow

Alexis Bougoin





Constant flow rate: backward traveling frictional plug ?

Jamming rays

Franceso
Melo da Rocha