Direct numerical simulation of granular media and suspensions

Taking close interactions into account

Ecole 2022 du GdR MathGeoPhy

Aline Lefebvre - Lepot
28 octobre 2022

Suspensions and granular media.

Macroscopic, non-brownian entities

Rheology.

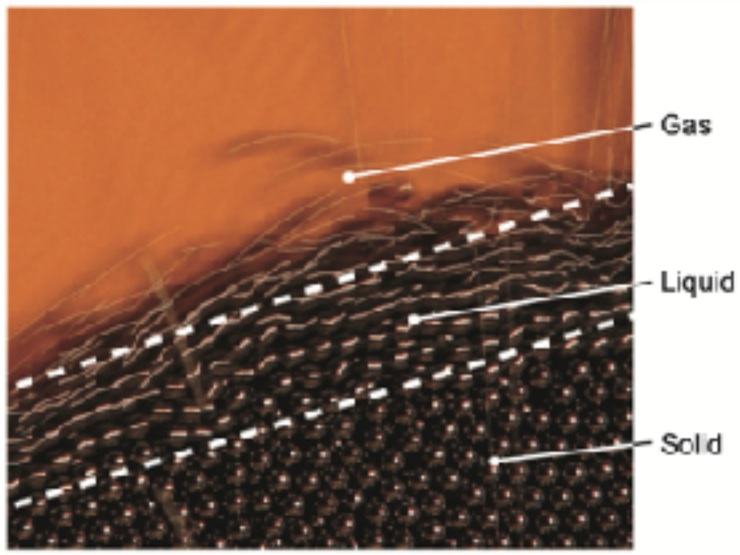
Rheology : ” the branch of physics concerned with
the flow and change of shape of matter ”

[Collins English Dictionary]

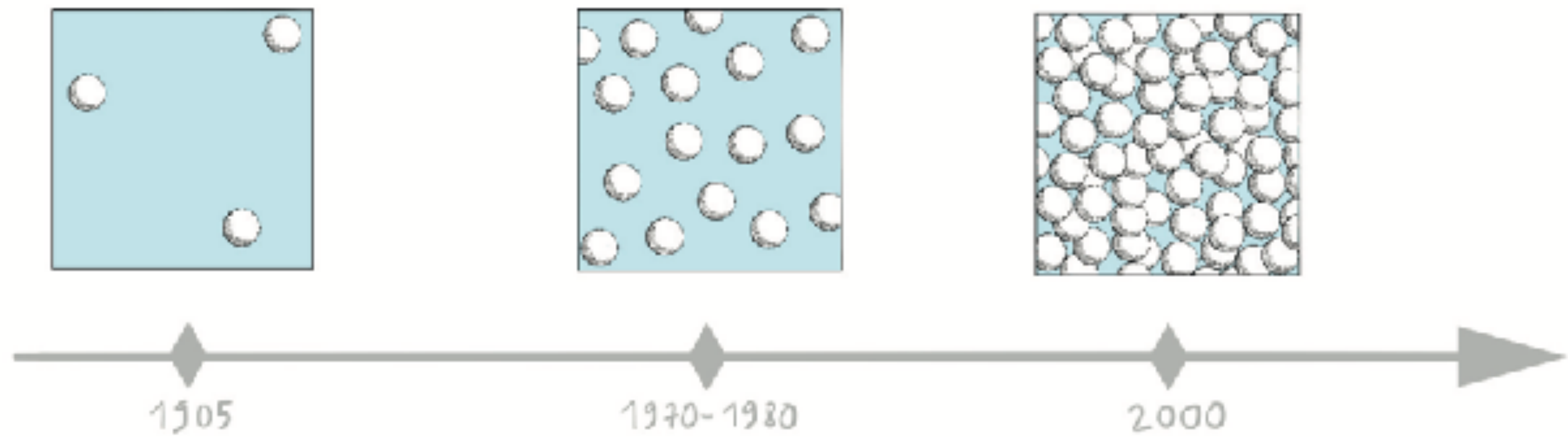
- flow, segregation, mixing, blocking, collapse...

Macroscopic behaviour

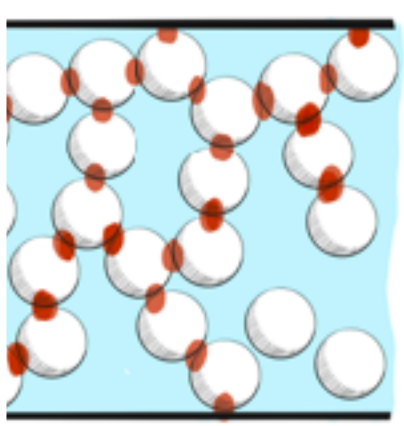
A complex behaviour!



B. Andreotti, Y. Forterre, and O. Pouliquen. Granular media: between fluid and solid. Cambridge University Press, 2013



E. Guazzelli and O. Pouliquen. “Rheology of dense granular suspensions”. In: Journal of Fluid Mechanics 852 (2018)



Multi-particle interactions

Contact Friction

Shape of particles

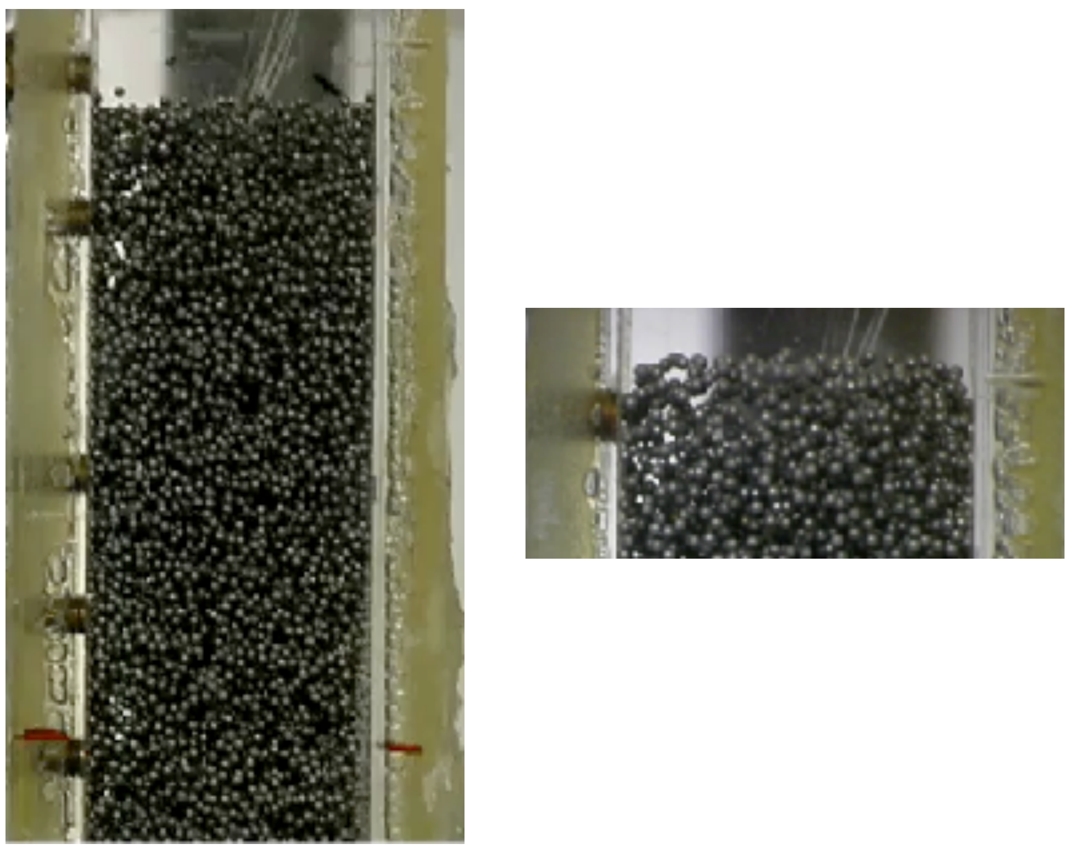
Local micro-structure

Non-local effects

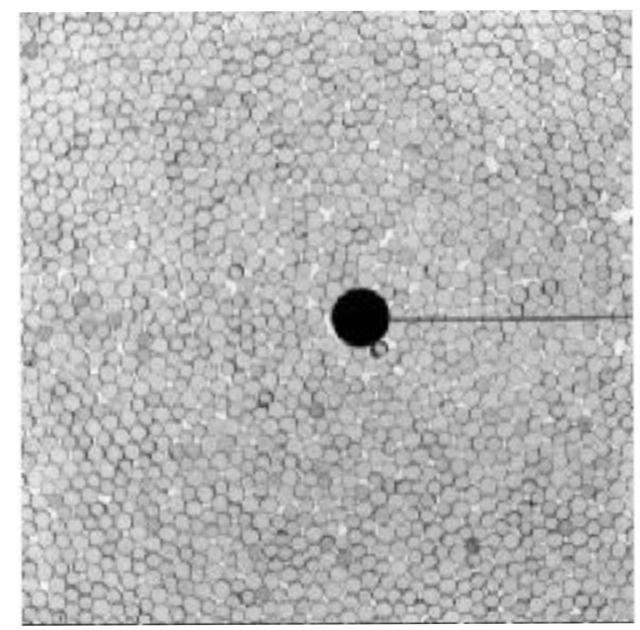
- Active domain of research
- Need for numerical simulations

Numerical simulations. The difficulties.

- Macroscopic behaviour $\Rightarrow$ Large number of particles
- Steady state and time average $\Rightarrow$ Long time simulations



G. Gauthier, J. Martin, FAST laboratory.

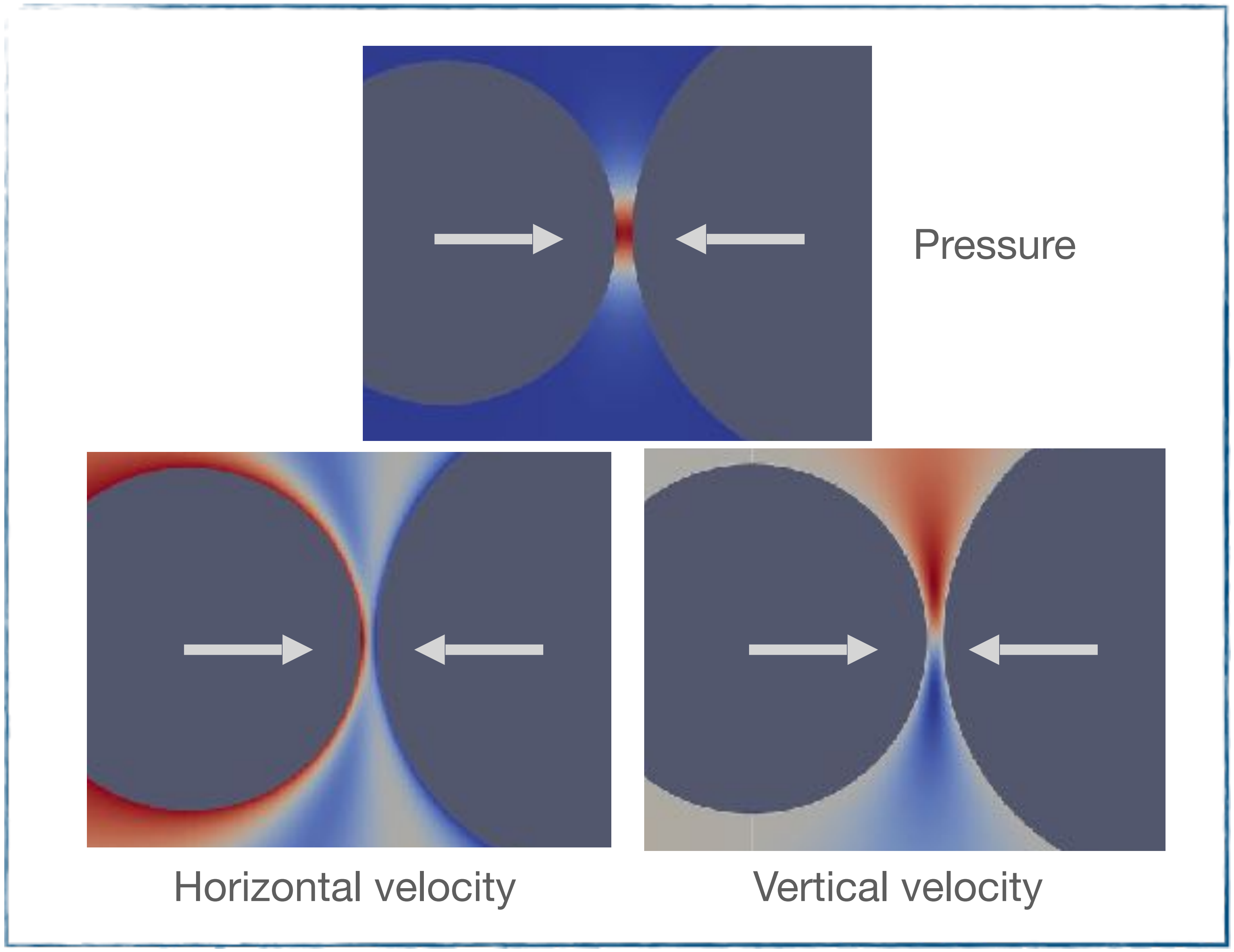


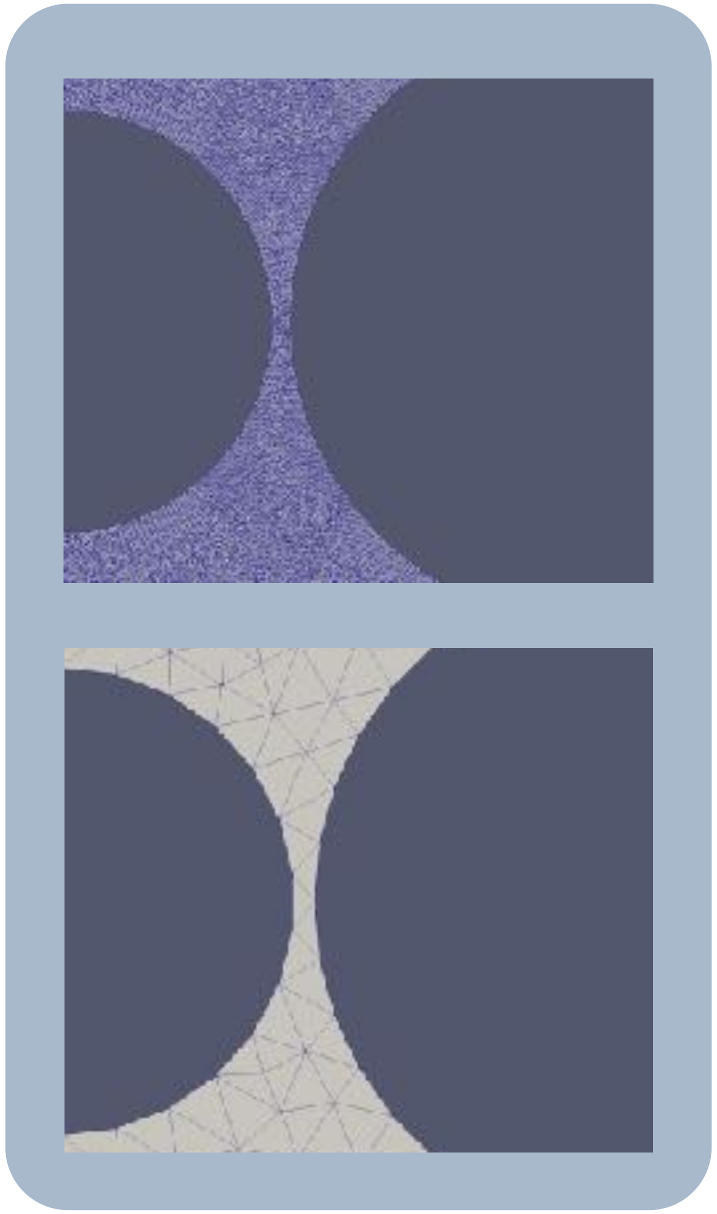
A. Seguin, P. Gondret, FAST laboratory.

Need for fast N-body computations

Numerical simulations. The difficulties.

- Dense suspensions ⇒ Close interactions due to the fluid





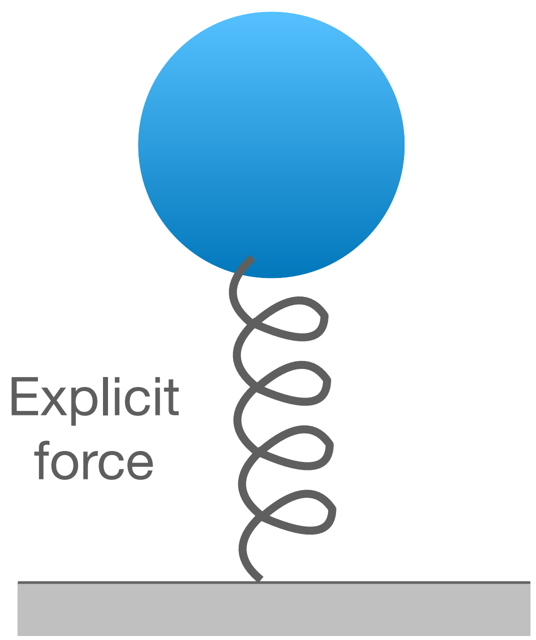
Need for methods taking lubrication into account

Numerical simulations. The difficulties.

- Dense suspensions
- Granular media

$\Rightarrow$ Solid contacts between particles

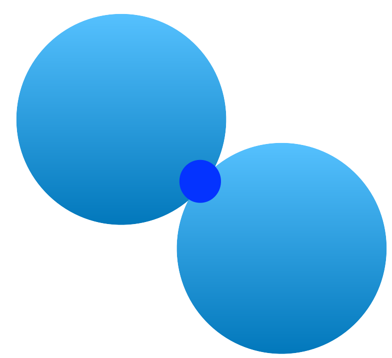
Molecular Dynamics



Continuous penalization method

[Cundall, Strack, 1979]

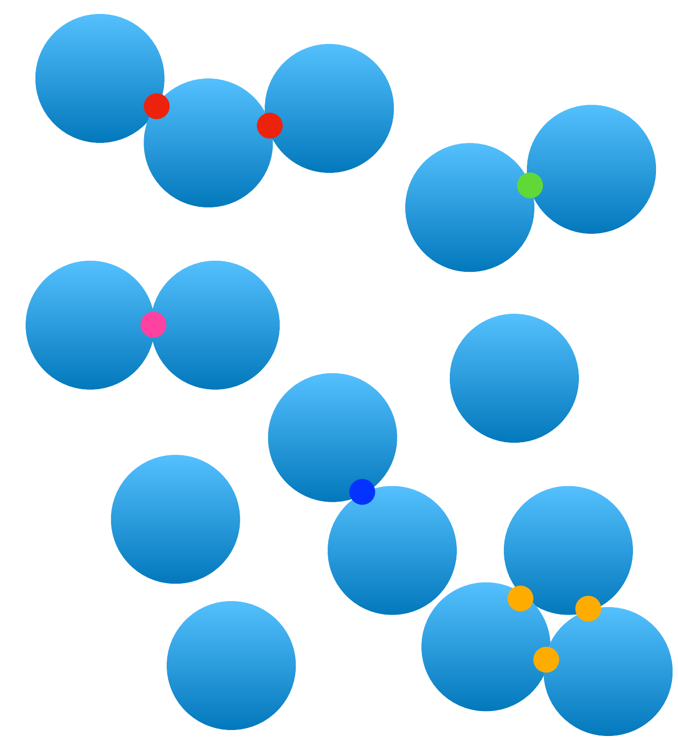
Contact law



Inelastic contacts
Coulomb friction law

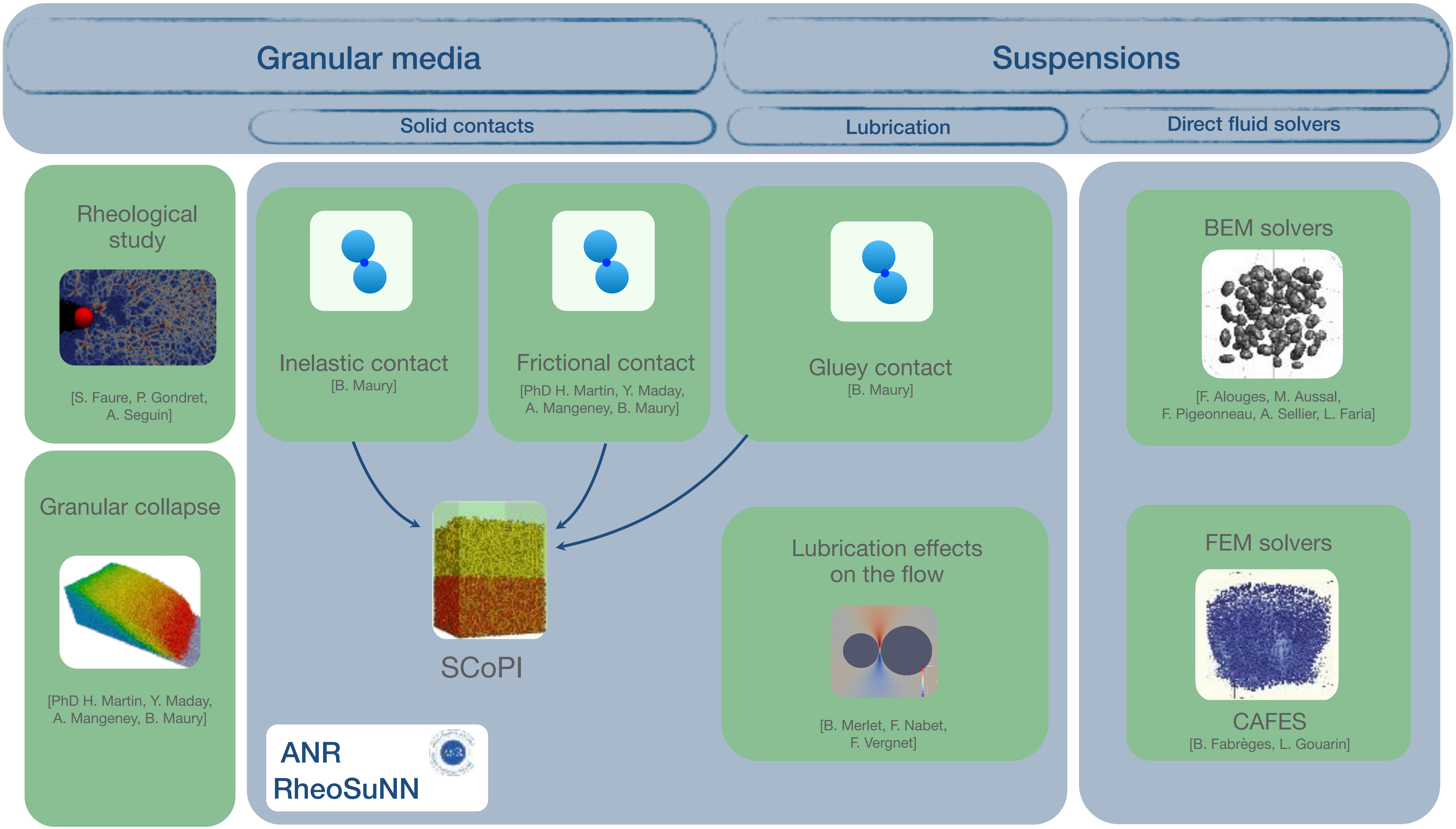
Explicit solution for 2 particles

Multi-contact problem

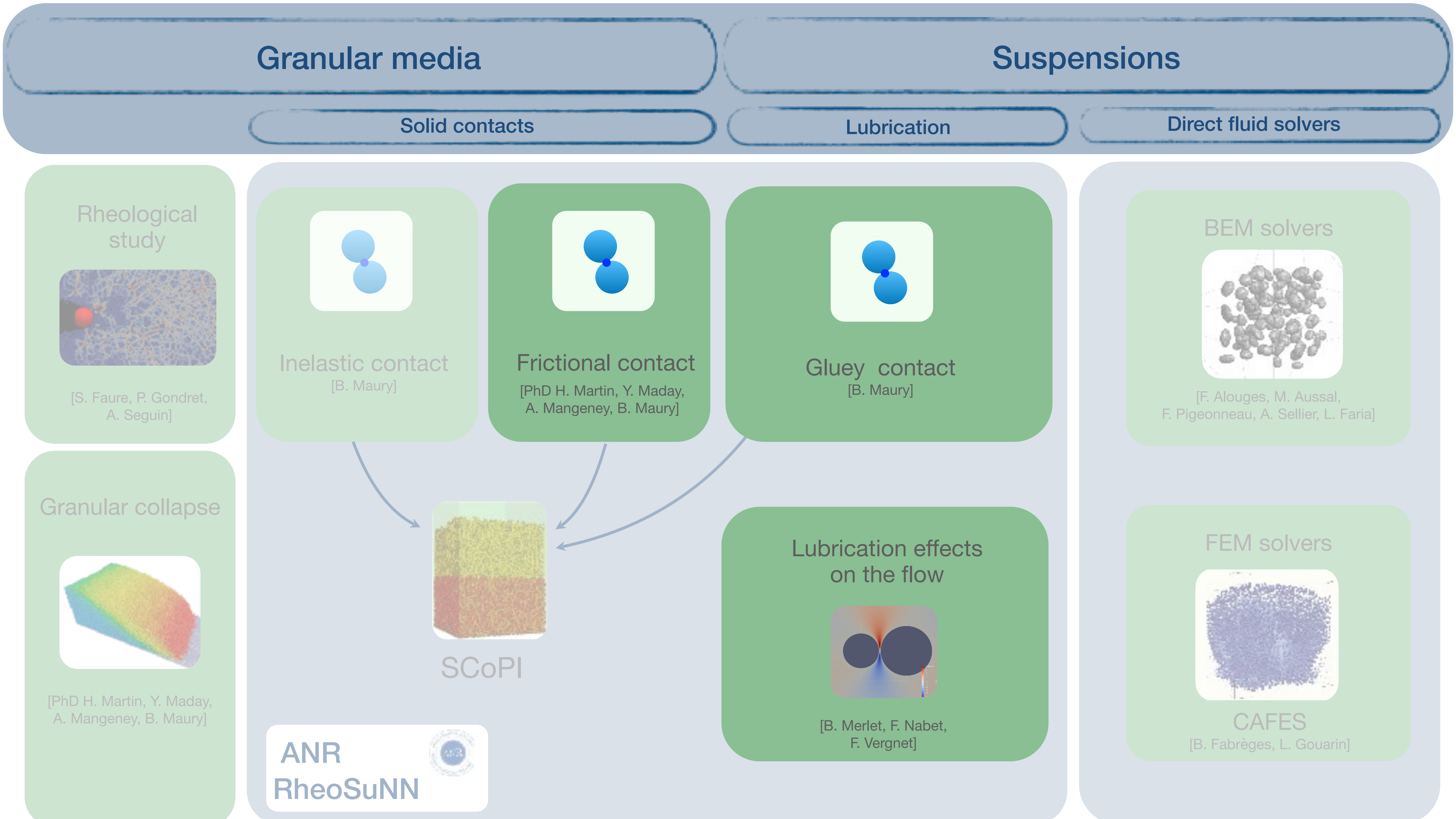


Need for a stable algorithm to deal with contacts

Overview.

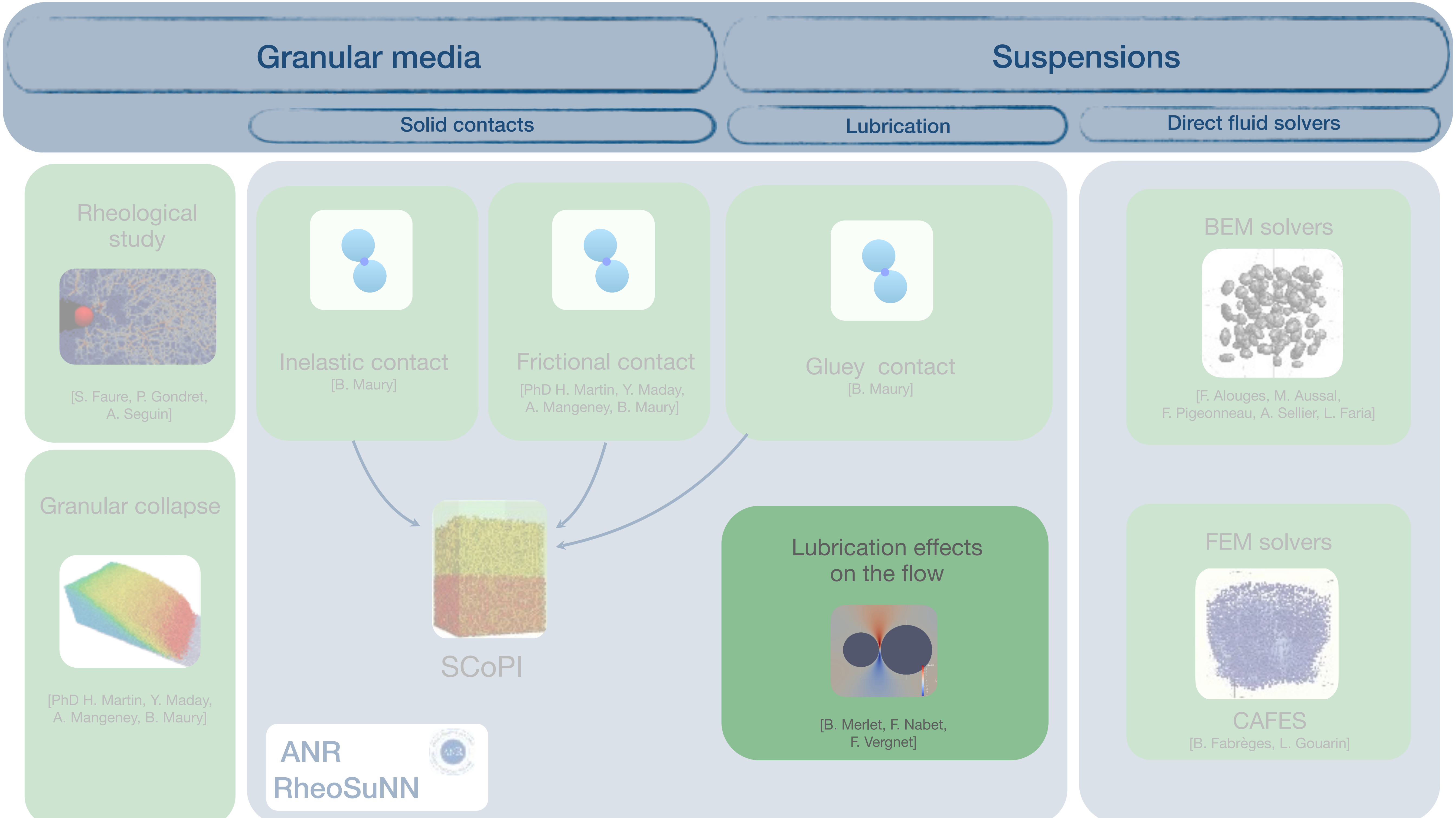

Granular media
Suspensions
Solid contacts
Lubrication
Direct fluid solvers
Rheological study
[S. Faure, P. Gondret, A. Seguin]
Granular collapse
[PhD H. Martin, Y. Maday, A. Mangeney, B. Maury]
Inelastic contact
[B. Maury]
Frictional contact
[PhD H. Martin, Y. Maday, A. Mangeney, B. Maury]
Gluey contact
[B. Maury]
SCoPI
Lubrication effects on the flow
[B. Merlet, F. Nabet, F. Vergnet]
ANR RheoSuNN
BEM solvers
[F. Alouges, M. Aussal, F. Pigeonneau, A. Sellier, L. Faria]
FEM solvers
CAFES
[B. Fabrèges, L. Gouarin]

Overview.

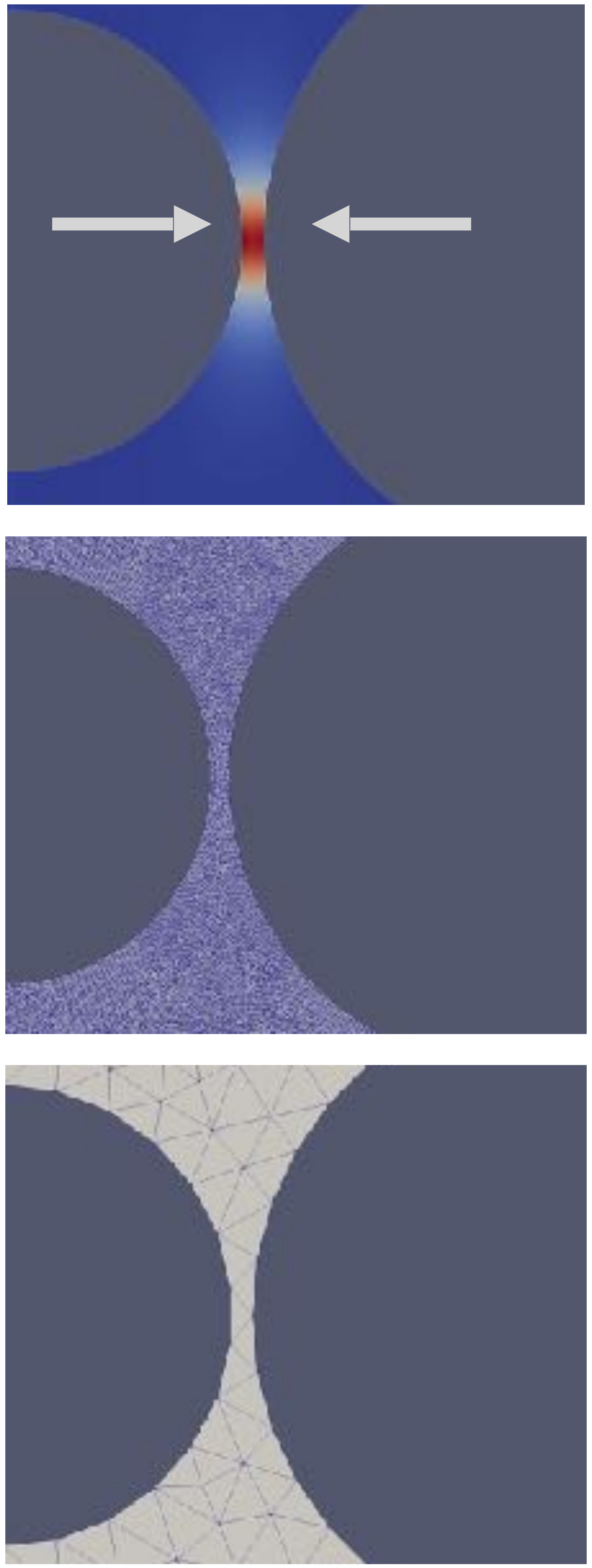


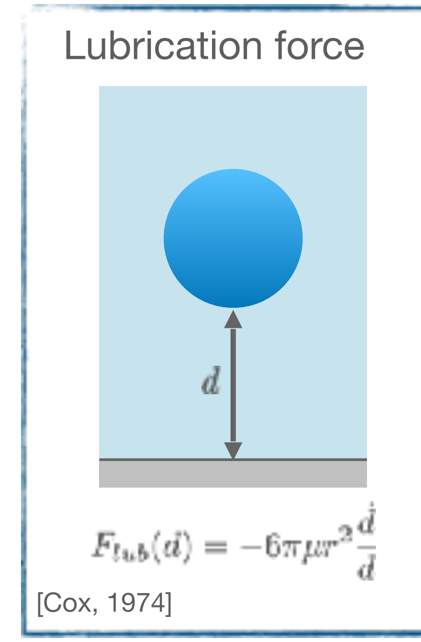
Granular media
Suspensions
Solid contacts
Lubrication
Direct fluid solvers
Rheological study
[S. Faure, P. Gondret, A. Seguin]
Granular collapse
[PhD H. Martin, Y. Maday, A. Mangeney, B. Maury]
Inelastic contact
[B. Maury]
Frictional contact
[PhD H. Martin, Y. Maday, A. Mangeney, B. Maury]
Gluey contact
[B. Maury]
SCoPI
Lubrication effects on the flow
[B. Merlet, F. Nabet, F. Vergnet]
ANR RheoSuNN
BEM solvers
[F. Alouges, M. Aussal, F. Pigeonneau, A. Sellier, L. Faria]
FEM solvers
CAFES
[B. Fabrèges, L. Gouarin]

Taking lubrication into account in the fluid solver.



Taking lubrication into account in the fluid solver.

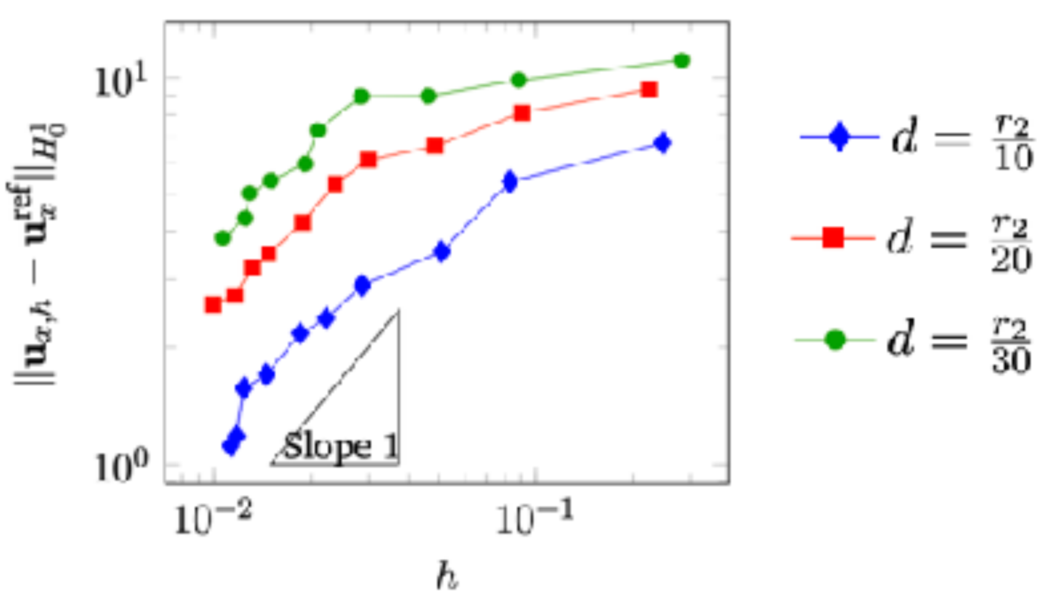




- A singular problem
  - Correction of the flow?
  - Non-spherical particles?

$$
\begin{aligned}
-\mu\Delta\mathbf{u} + \nabla\mathrm{p} = 0 &\quad \text{in } \mathcal{F}\\
\nabla\cdot\mathbf{u} = 0 &\quad \text{in } \mathcal{F}\\
\mathbf{u} = \mathbf{u}^\star &\quad \text{on } \partial B
\end{aligned}
$$

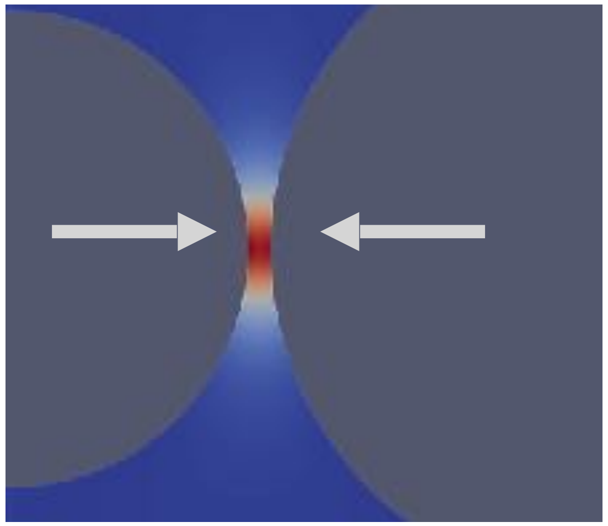
$$\|\mathbf{u} - \mathbf{u}_h\| \leq C\,\|(\mathbf{u},\mathrm{p})\|\,h^\alpha$$

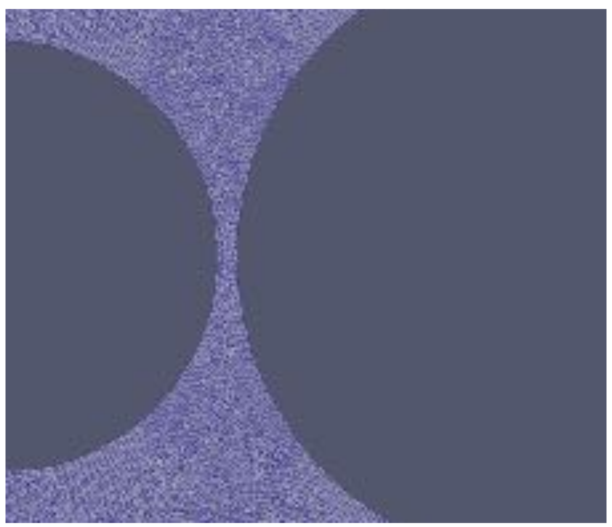


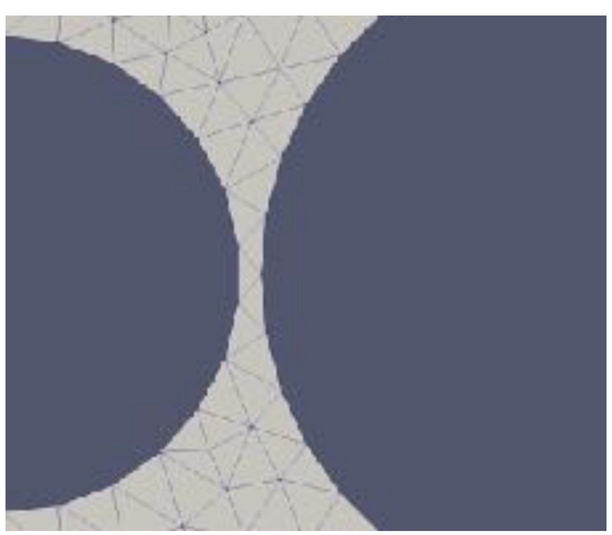
Blows up when the distance goes to zero…

An accurate method to include lubrication forces in numerical simulations of dense suspensions. With B. Merlet, and T. N. Nguyen. JFM, 769 (2015)
Numerical simulation of rigid particles in Stokes flow: lubrication correction for general shape of particles. With F. Nabet. Math. MMNP, 16 (2021)

Taking lubrication into account in the fluid solver.



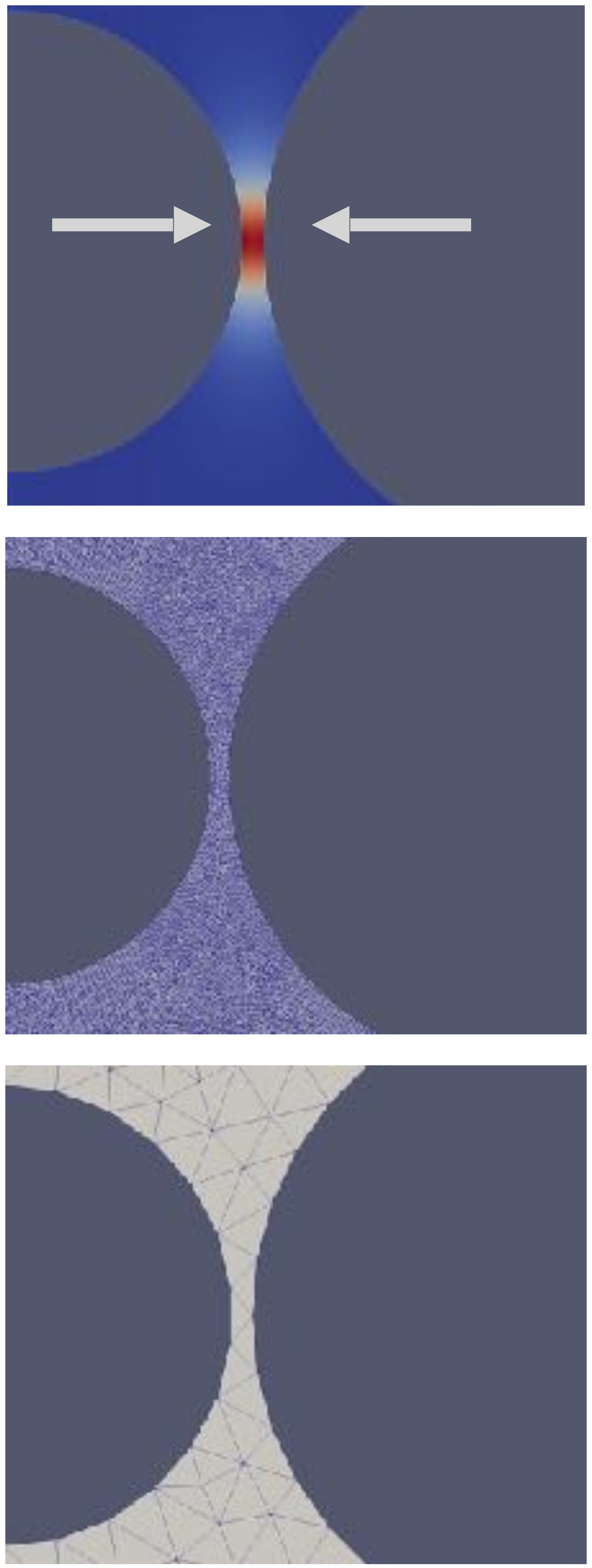




$$(\mathbf{u}_h^{\mathrm{new}}, \mathrm{p}_h^{\mathrm{new}}) = (\mathbf{u}^{\mathrm{sing}}, \mathrm{p}^{\mathrm{sing}}) + (\mathbf{u}_h^{\mathrm{reg}}, \mathrm{p}_h^{\mathrm{reg}})$$

An accurate method to include lubrication forces in numerical simulations of dense suspensions. With B. Merlet, and T. N. Nguyen. JFM, 769 (2015)
Numerical simulation of rigid particles in Stokes flow: lubrication correction for general shape of particles. With F. Nabet. Math. MMNP, 16 (2021)

Taking lubrication into account in the fluid solver.

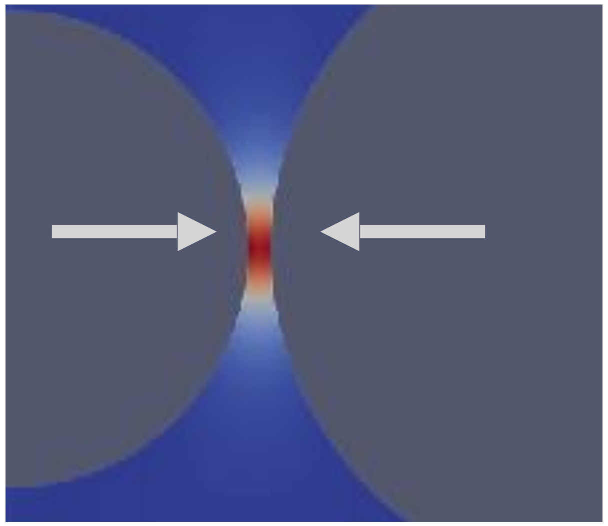


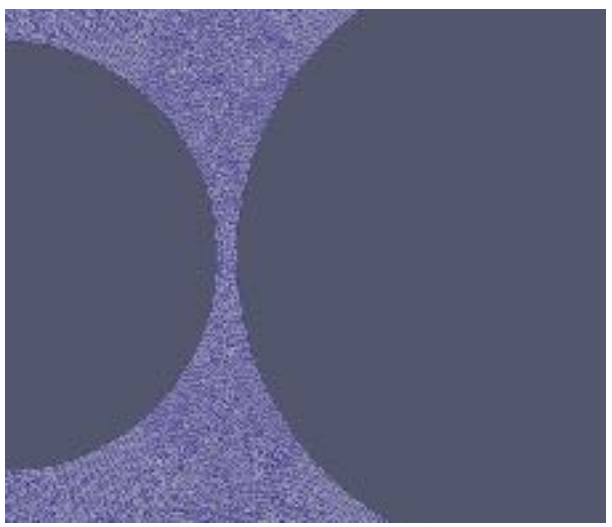
Explicit asymptotic expansion

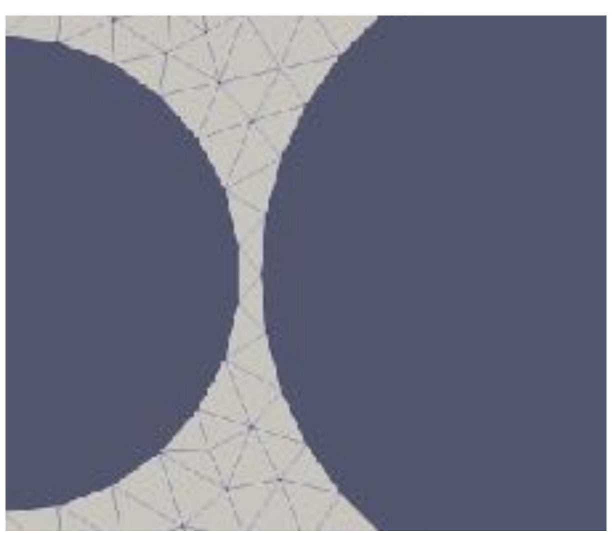
$$(\mathbf{u}_h^{\text{new}}, \mathrm{p}_h^{\text{new}}) = (\mathbf{u}^{\text{sing}}, \mathrm{p}^{\text{sing}}) + (\mathbf{u}_h^{\text{reg}}, \mathrm{p}_h^{\text{reg}})$$

An accurate method to include lubrication forces in numerical simulations of dense suspensions. With B. Merlet, and T. N. Nguyen. JFM, 769 (2015)
Numerical simulation of rigid particles in Stokes flow: lubrication correction for general shape of particles. With F. Nabet. Math. MMNP, 16 (2021)

Taking lubrication into account in the fluid solver.







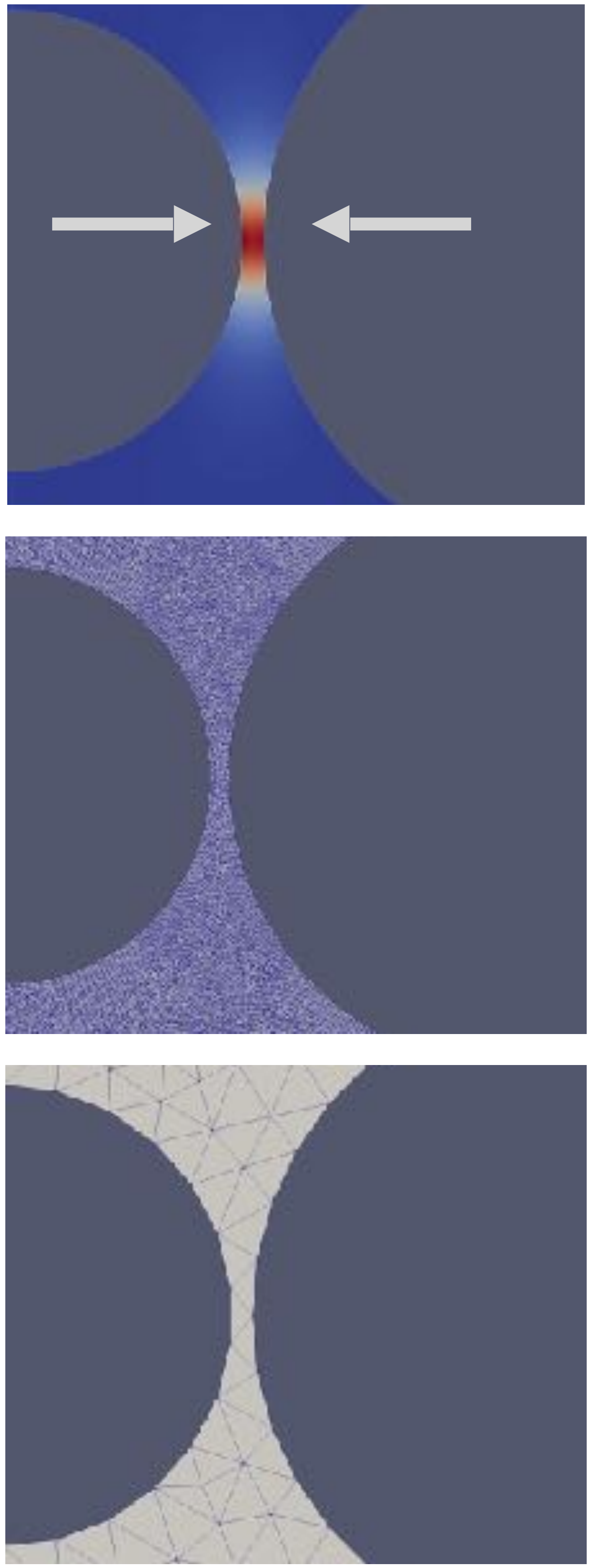
Explicit asymptotic expansion

$$(\mathbf{u}_h^{\mathrm{new}}, \mathrm{p}_h^{\mathrm{new}}) = (\mathbf{u}^{\mathrm{sing}}, \mathrm{p}^{\mathrm{sing}}) + (\mathbf{u}_h^{\mathrm{reg}}, \mathrm{p}_h^{\mathrm{reg}})$$

$$\begin{aligned} -\mu\triangle\mathbf{u}^{\mathrm{reg}} + \nabla\mathrm{p}^{\mathrm{reg}} &= \mu\triangle\mathbf{u}^{\mathrm{sing}} - \nabla\mathrm{p}^{\mathrm{sing}} && \text{in } \mathcal{F} \\ \nabla\cdot\mathbf{u}^{\mathrm{reg}} &= -\nabla\cdot\mathbf{u}^{\mathrm{sing}} && \text{in } \mathcal{F} \\ \mathbf{u}^{\mathrm{reg}} &= \mathbf{u}^{\star} - \mathbf{u}^{\mathrm{sing}} && \text{on } \partial B \end{aligned}$$

An accurate method to include lubrication forces in numerical simulations of dense suspensions. With B. Merlet, and T. N. Nguyen. JFM, 769 (2015)
Numerical simulation of rigid particles in Stokes flow: lubrication correction for general shape of particles. With F. Nabet. Math. MMNP, 16 (2021)

Taking lubrication into account in the fluid solver.



Explicit asymptotic expansion

$$(\mathbf{u}_h^{\mathrm{new}}, \mathrm{p}_h^{\mathrm{new}}) = (\mathbf{u}^{\mathrm{sing}}, \mathrm{p}^{\mathrm{sing}}) + (\mathbf{u}_h^{\mathrm{reg}}, \mathrm{p}_h^{\mathrm{reg}})$$

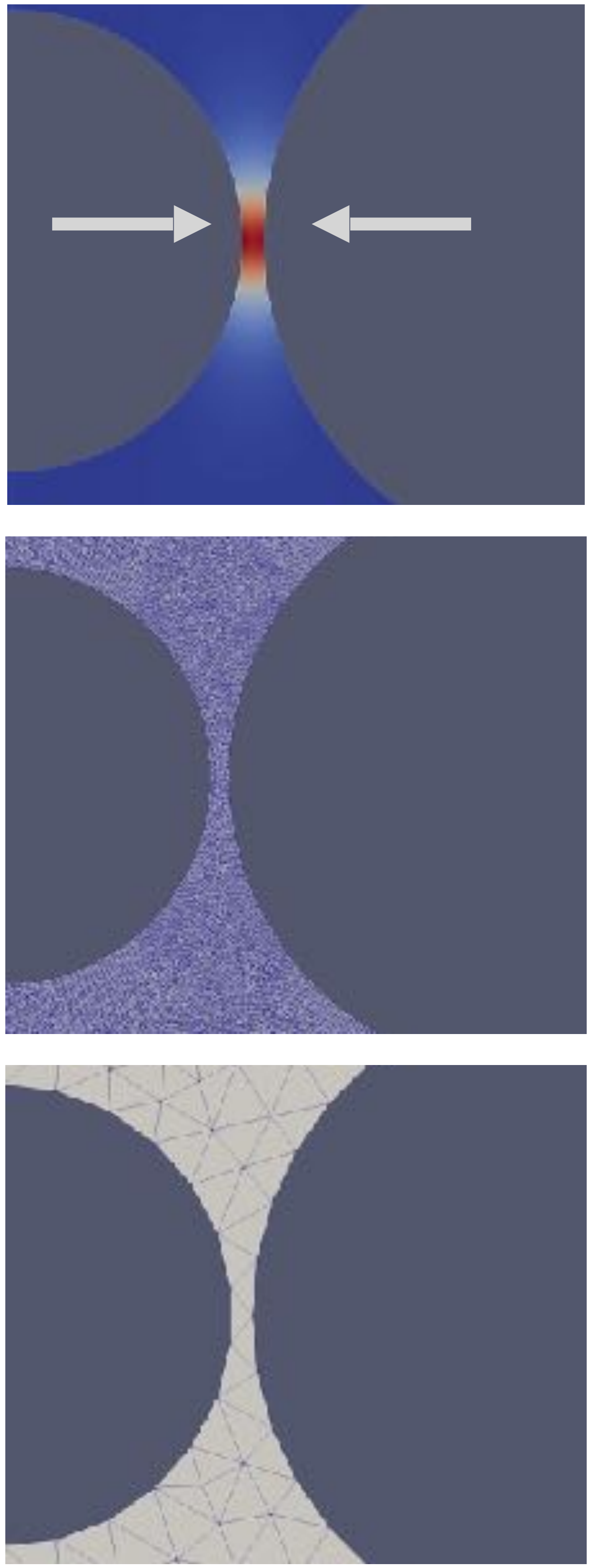
$$\begin{aligned} -\mu\triangle\mathbf{u}^{\mathrm{reg}} + \nabla\mathrm{p}^{\mathrm{reg}} &= \mu\triangle\mathbf{u}^{\mathrm{sing}} - \nabla\mathrm{p}^{\mathrm{sing}} && \text{in } \mathcal{F} \\ \nabla\cdot\mathbf{u}^{\mathrm{reg}} &= -\nabla\cdot\mathbf{u}^{\mathrm{sing}} && \text{in } \mathcal{F} \\ \mathbf{u}^{\mathrm{reg}} &= \mathbf{u}^{\star} - \mathbf{u}^{\mathrm{sing}} && \text{on } \partial B \end{aligned}$$

$$\|\mathbf{u} - \mathbf{u}_h^{\mathrm{new}}\| \leq C\,\|(\mathbf{u}^{\mathrm{reg}}, \mathrm{p}^{\mathrm{reg}})\|\, h^{\alpha}$$

Bounded independently of the distance

An accurate method to include lubrication forces in numerical simulations of dense suspensions. With B. Merlet, and T. N. Nguyen. JFM, 769 (2015)
Numerical simulation of rigid particles in Stokes flow: lubrication correction for general shape of particles. With F. Nabet. Math. MMNP, 16 (2021)

Taking lubrication into account in the fluid solver.

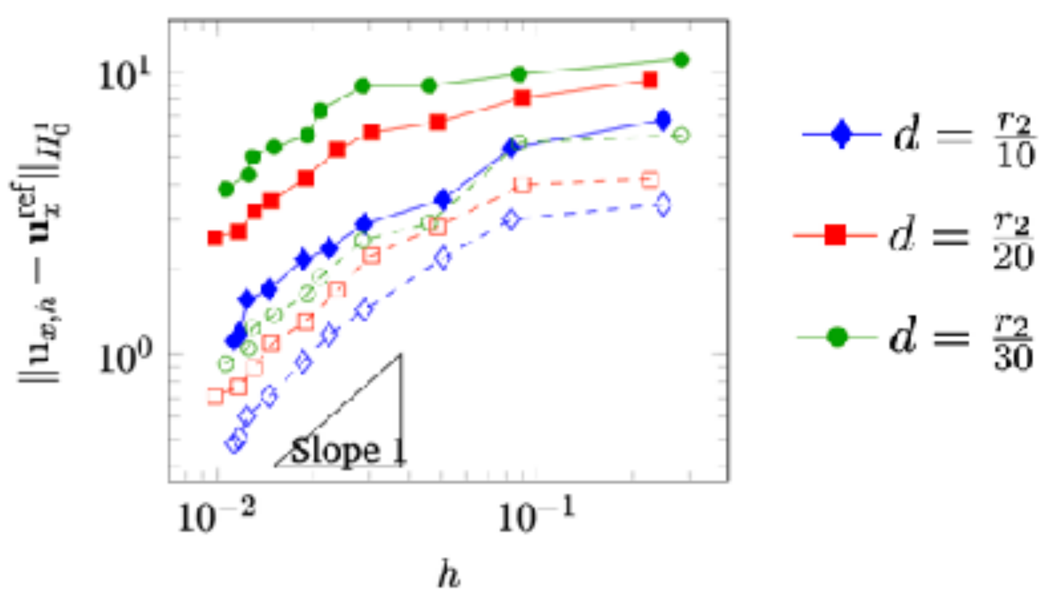


Explicit asymptotic expansion

$$(\mathbf{u}_h^{\mathrm{new}}, \mathrm{p}_h^{\mathrm{new}}) = (\mathbf{u}^{\mathrm{sing}}, \mathrm{p}^{\mathrm{sing}}) + (\mathbf{u}_h^{\mathrm{reg}}, \mathrm{p}_h^{\mathrm{reg}})$$

$$\begin{aligned} -\mu\triangle\mathbf{u}^{\mathrm{reg}} + \nabla\mathrm{p}^{\mathrm{reg}} &= \mu\triangle\mathbf{u}^{\mathrm{sing}} - \nabla\mathrm{p}^{\mathrm{sing}} && \text{in } \mathcal{F} \\ \nabla\cdot\mathbf{u}^{\mathrm{reg}} &= -\nabla\cdot\mathbf{u}^{\mathrm{sing}} && \text{in } \mathcal{F} \\ \mathbf{u}^{\mathrm{reg}} &= \mathbf{u}^{\star} - \mathbf{u}^{\mathrm{sing}} && \text{on } \partial B \end{aligned}$$

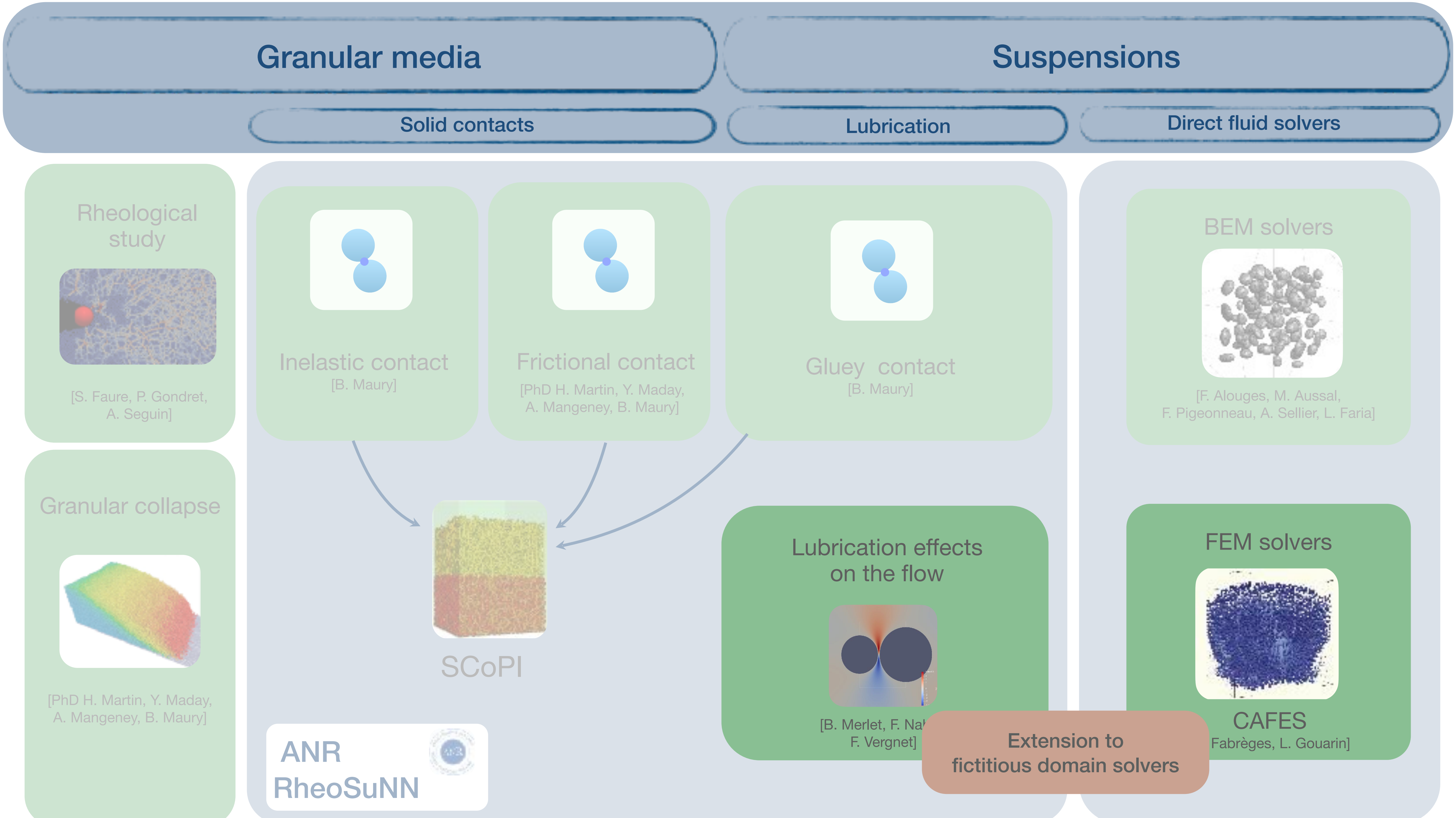
$$\|\mathbf{u} - \mathbf{u}_h^{\mathrm{new}}\| \leq C\,\|(\mathbf{u}^{\mathrm{reg}}, \mathrm{p}^{\mathrm{reg}})\|\, h^{\alpha}$$



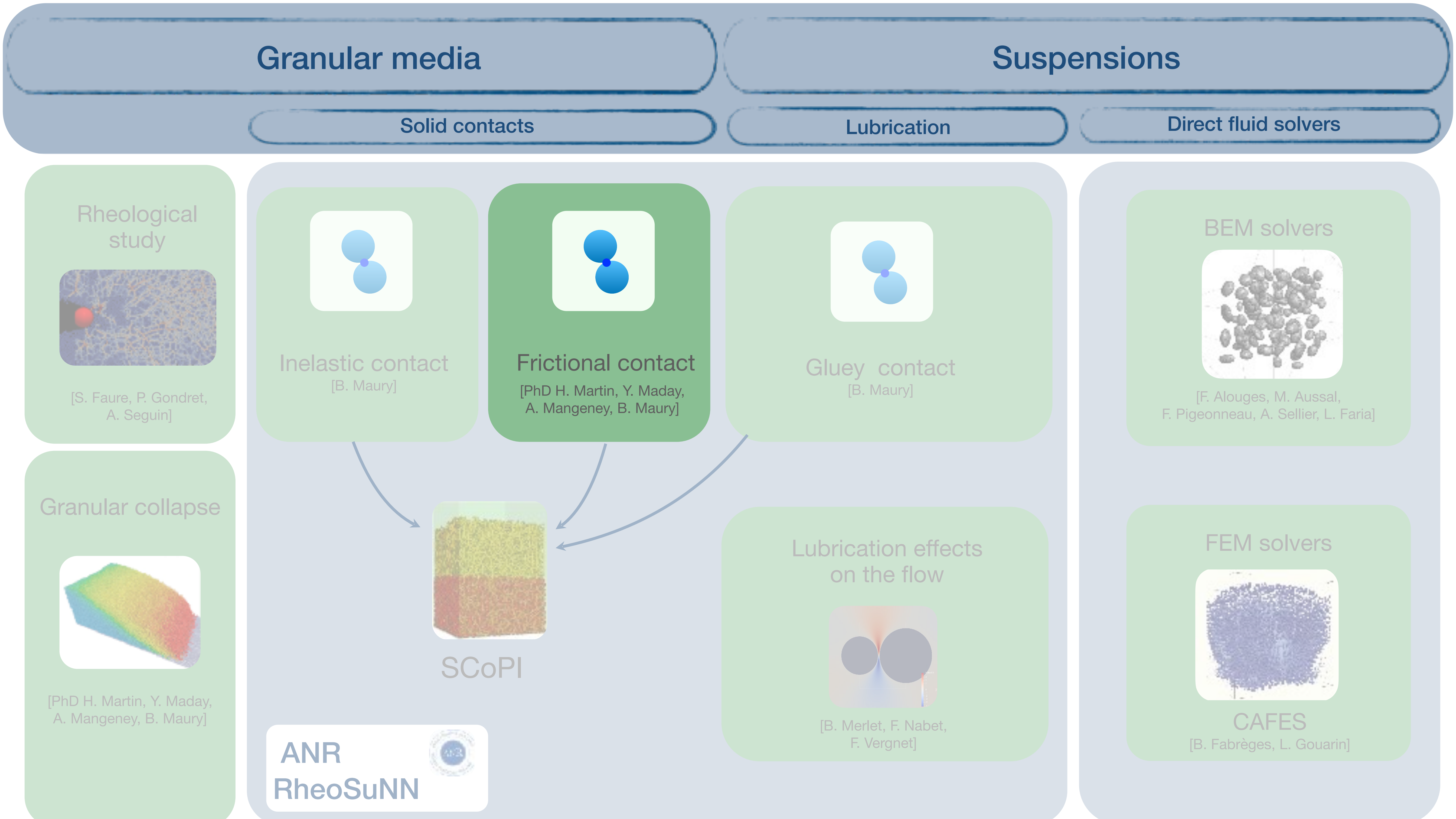
Bounded independently of the distance

An accurate method to include lubrication forces in numerical simulations of dense suspensions. With B. Merlet, and T. N. Nguyen. JFM, 769 (2015)
Numerical simulation of rigid particles in Stokes flow: lubrication correction for general shape of particles. With F. Nabet. Math. MMNP, 16 (2021)

Taking lubrication into account in the fluid solver.

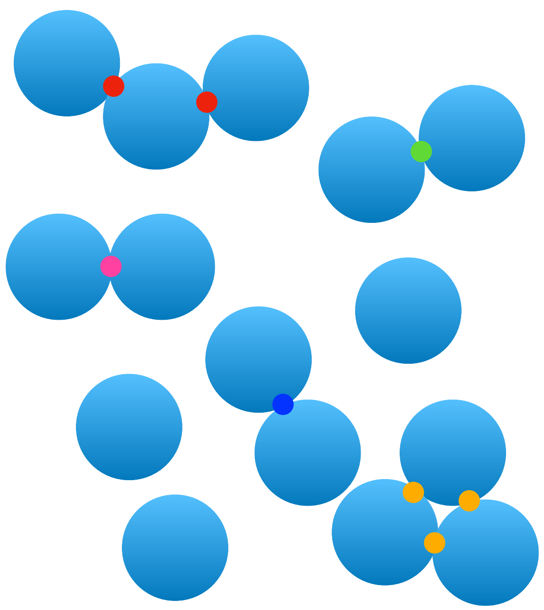


Stable and convex contact algorithms.



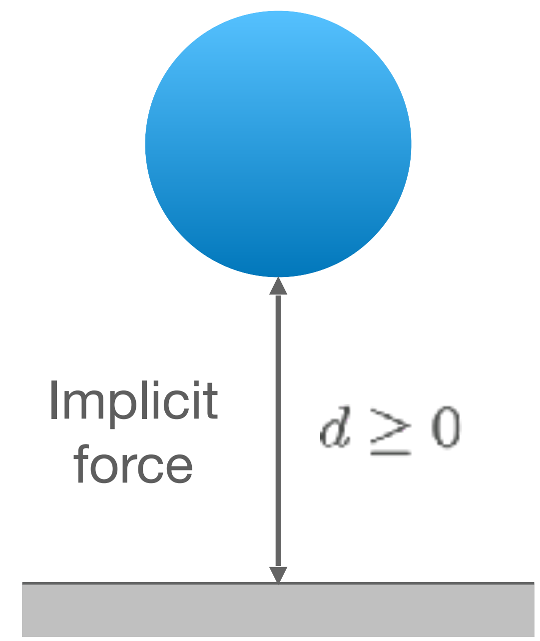
Stable and convex contact algorithms.

Multi-contact problem



Inelastic contacts
Coulomb law

Non-Smooth
Contact Dynamics



Non-smooth convex
analysis

[Moreau, Jean, 1992-1999]

Stable and convex contact algorithms.

Inelastic contact

Inelastic contact with friction

$$\min_{\mathbf{v}\in K} \frac{1}{2}\left\|\mathbf{v} - \mathbf{V}^{k+1}\right\|_M$$

$$K = \left\{\mathbf{v},\ D^k + dt\nabla D^k \cdot \mathbf{v} \geq 0\right\}$$

Linear constraint

SCoPI

Existence results for non-smooth second order differential inclusions…With F. Bernicot. In: Confluentes Mathematici 2.4 (2010), pp. 445–471.
An optimization-based model for dry granular flows: application to granular collapse on erodible beds, with H. Martin, A. Mangeney, Y. Maday, B. Maury, Submitted

Stable and convex contact algorithms.

Inelastic contact

Inelastic contact with friction

$$\min_{\mathbf{v}\in K} \frac{1}{2}\left\|\mathbf{v}-\mathbf{V}^{k+1}\right\|_M$$

$$K = \{\mathbf{v},\ D^k + dt\nabla D^k\cdot\mathbf{v}\geq 0\}$$

Linear constraint

SCoPI

$$m\frac{\mathbf{v}^{k+1}-\mathbf{v}^k}{dt} = \mathbf{f}^{\text{ext}} + f_n\mathbf{n}^k$$

$$D^k + dt\nabla D^k\cdot\mathbf{v}^{k+1}\geq 0$$

$$f_n \geq 0\ +\ \text{complem.}$$

[Maury, 2006]

Existence results for non-smooth second order differential inclusions…With F. Bernicot. In: Confluentes Mathematici 2.4 (2010), pp. 445–471.
An optimization-based model for dry granular flows: application to granular collapse on erodible beds, with H. Martin, A. Mangeney, Y. Maday, B. Maury, Submitted

Stable and convex contact algorithms.

Inelastic contact

Inelastic contact with friction

$$\min_{\mathbf{v}\in K} \frac{1}{2}\left\|\mathbf{v}-\mathbf{V}^{k+1}\right\|_M$$

$$K=\left\{\mathbf{v},\ D^k+dt\nabla D^k\cdot\mathbf{v}\geq 0\right\}$$

Linear constraint

Conic constraint

$$\min_{\mathbf{v}\in K_\mu} \frac{1}{2}\left\|\mathbf{u}-\mathbf{U}^{k+1}\right\|_M$$

$$K_\mu=\left\{\mathbf{u},\ D^k+dt\nabla D^k\cdot\mathbf{v} \geq \mu\, dt\, |T\mathbf{u}|\ \right\}$$

SCoPI

[H. Martin]

$$m\frac{\mathbf{v}^{k+1}-\mathbf{v}^k}{dt}=\mathbf{f}^{\text{ext}}+f_n\mathbf{n}^k$$

$$D^k+dt\nabla D^k\cdot\mathbf{v}^{k+1}\geq 0$$

$$f_n\geq 0\ +\ \text{complem.}$$

[Maury, 2006]

Existence results for non-smooth second order differential inclusions…With F. Bernicot. In: Confluentes Mathematici 2.4 (2010), pp. 445–471.
An optimization-based model for dry granular flows: application to granular collapse on erodible beds, with H. Martin, A. Mangeney, Y. Maday, B. Maury, Submitted

Stable and convex contact algorithms.

Inelastic contact

$$\min_{\mathbf{v}\in K} \frac{1}{2} \left\| \mathbf{v} - \mathbf{V}^{k+1} \right\|_M$$

$$K = \{\mathbf{v},\ D^k + dt\nabla D^k \cdot \mathbf{v} \geq 0\}$$

Linear constraint

$$m\frac{\mathbf{v}^{k+1} - \mathbf{v}^k}{dt} = \mathbf{f}^{\text{ext}} + f_n \mathbf{n}^k$$

$$D^k + dt\nabla D^k \cdot \mathbf{v}^{k+1} \geq 0$$

$$f_n \geq 0 \ + \ \text{complem.}$$

[Maury, 2006]

Inelastic contact with friction

$$\min_{\mathbf{v}\in K_\mu} \frac{1}{2} \left\| \mathbf{u} - \mathbf{U}^{k+1} \right\|_M$$

$$K_\mu = \{\mathbf{u},\ D^k + dt\nabla D^k \cdot \mathbf{v} \geq \mu\, dt\, |T\mathbf{u}|\ \}$$

Conic constraint

$$m\frac{\mathbf{v}^{k+1} - \mathbf{v}^k}{dt} = \mathbf{f}^{\text{ext}} + (f_n \mathbf{n}^k + \mathbf{f}_t)$$

$$J\frac{\boldsymbol{\omega}^{k+1} - \boldsymbol{\omega}^k}{dt} = \mathbf{r} \wedge (f_n \mathbf{n}^k + \mathbf{f}_t)$$

$$D^k + dt\nabla D^k \cdot \mathbf{v}^{k+1} \geq \mu\, dt\, |T\mathbf{u}^{k+1}|$$

$$f_n \geq 0 \ + \ \text{complem.}$$

$$\text{Coulomb}\left(\mathbf{v}^{k+1}, \boldsymbol{\omega}^{k+1}, f_n, \mathbf{f}_t\right)$$

[Tassora, Negrut, Anitescu, 2008]

SCoPI

[H. Martin]

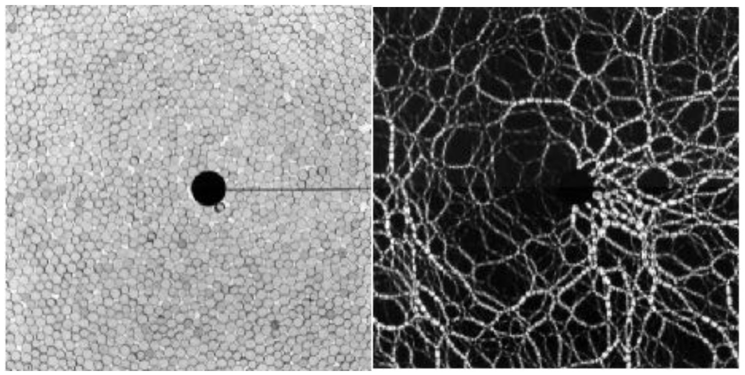
- Implicit algorithms
- No need to detect the contacts times
- Convex minimization problems at each time step

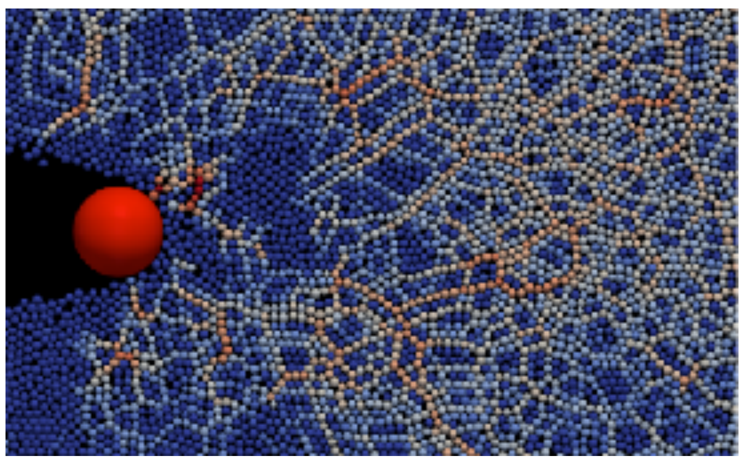
Existence results for non-smooth second order differential inclusions…With F. Bernicot. In: Confluentes Mathematici 2.4 (2010), pp. 445–471.

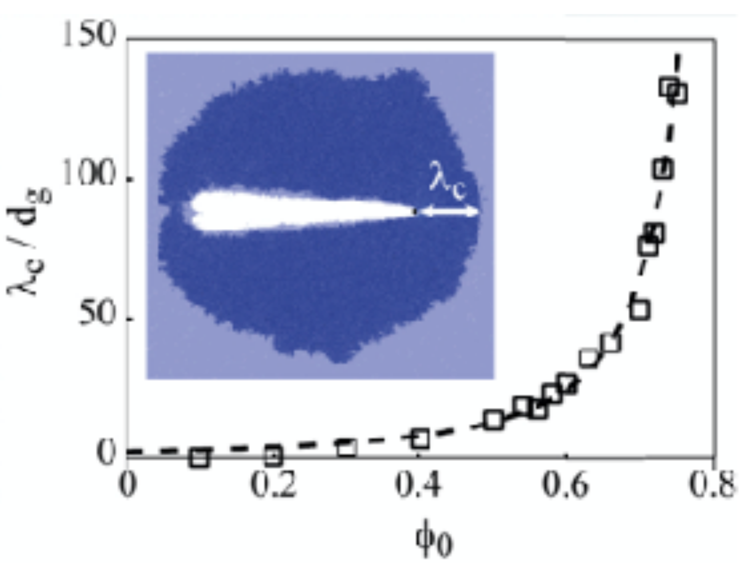
An optimization-based model for dry granular flows: application to granular collapse on erodible beds, with H. Martin, A. Mangeney, Y. Maday, B. Maury, Submitted

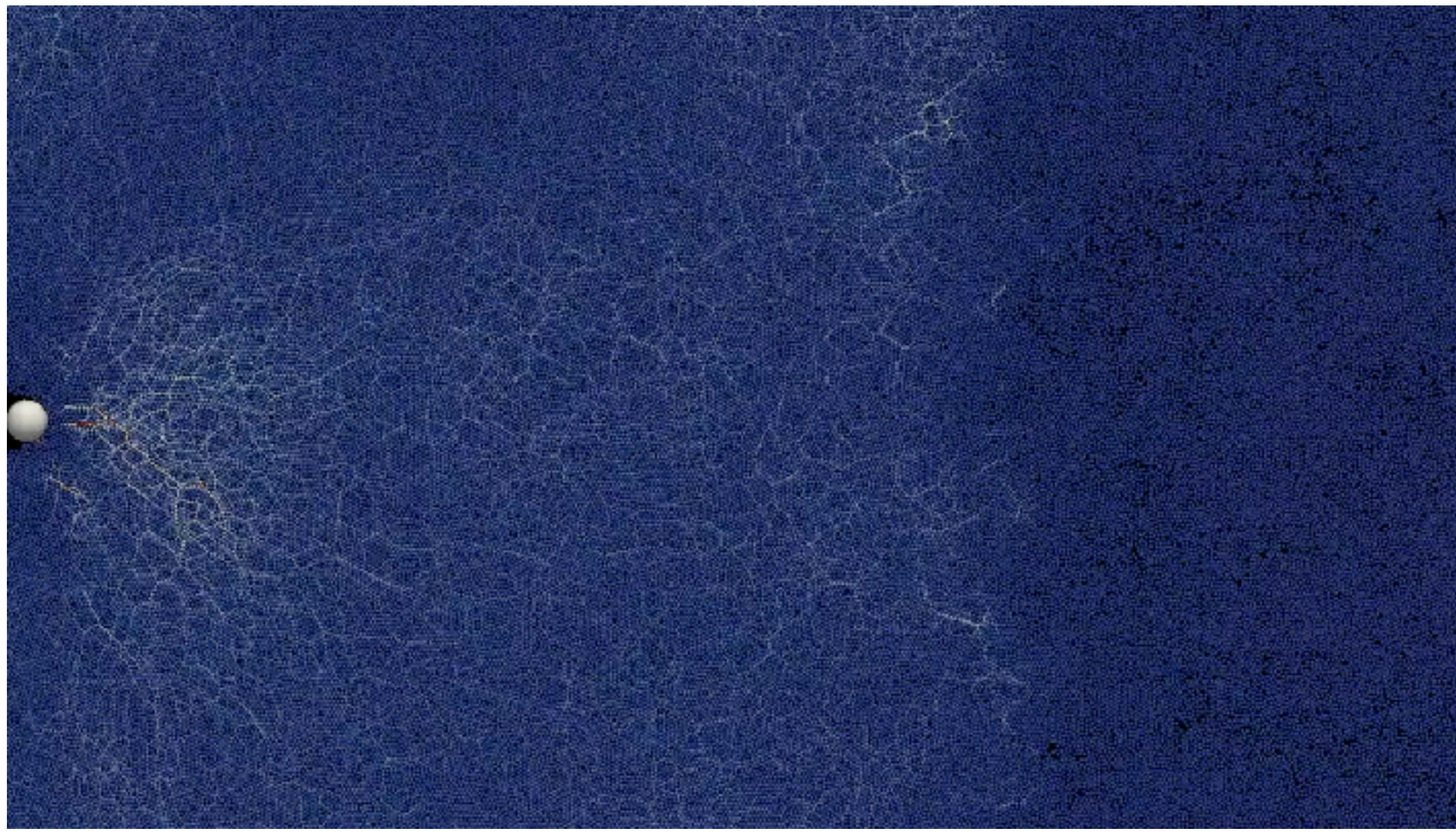
Stable and convex contact algorithms.

- Inelastic contacts
- Spherical particles
- SCoPI







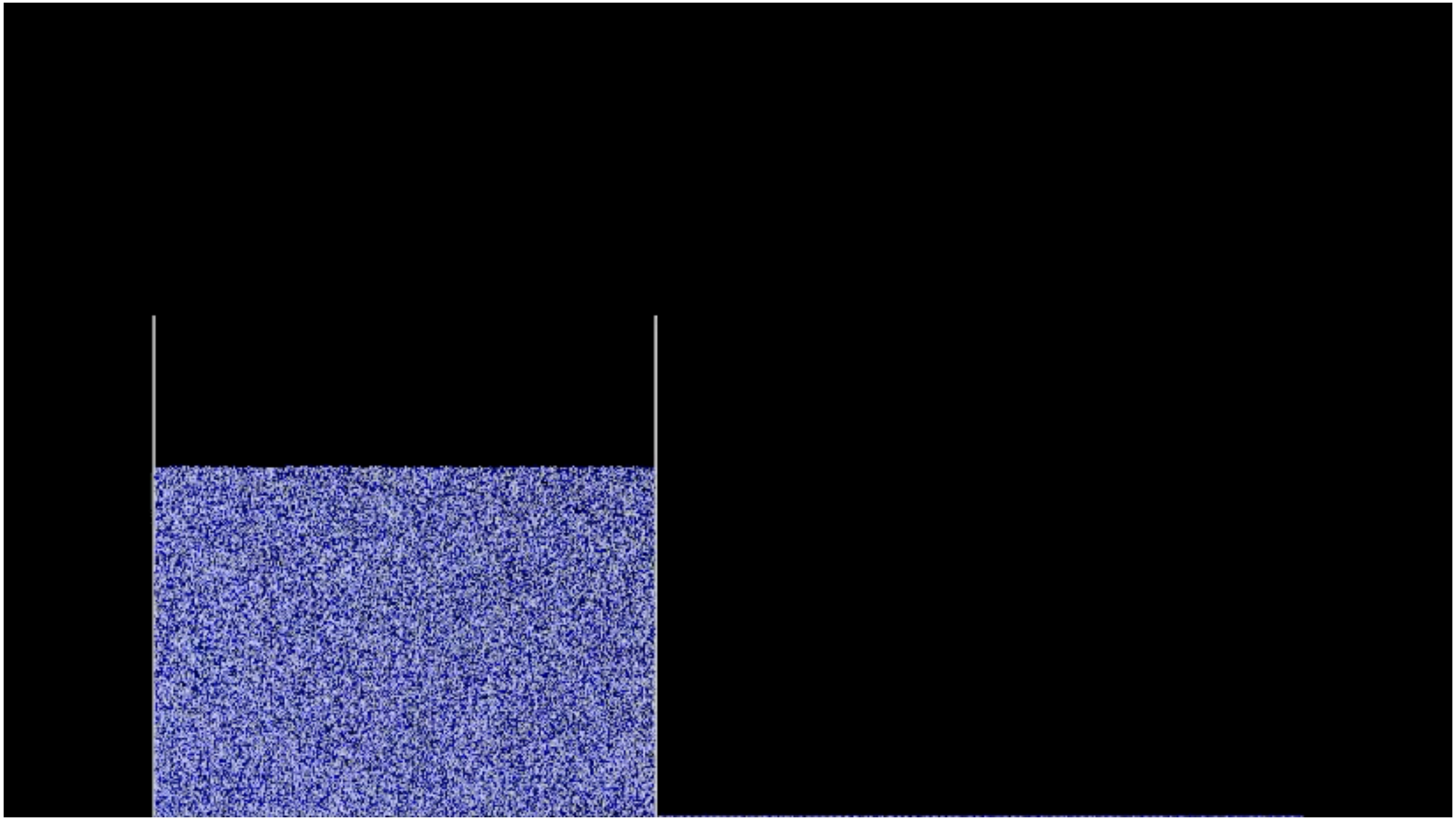


- Rheology
- Chains of forces

Clustering and flow around a sphere moving into a grain cloud. With A. Seguin, S. Faure, and P. Gondret.
In: The European Physical Journal E 39.6 (2016)

Stable and convex contact algorithms.

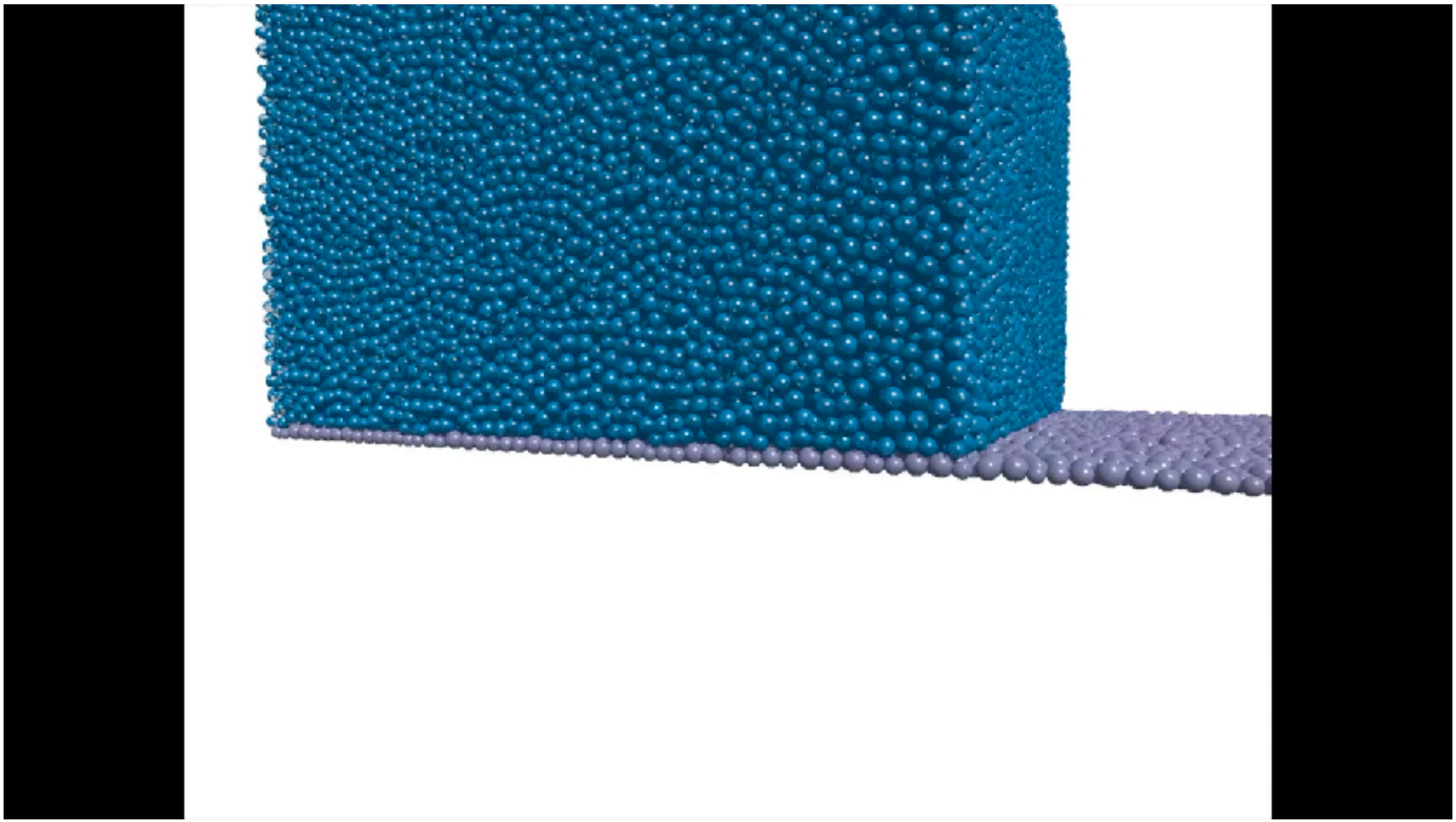
- Friction
- Spherical particles
- Hugo Martin [LJLL, IPGP]



An optimization-based model for dry granular flows: application to granular collapse on erodible beds, with H. Martin, A. Mangeney, Y. Maday, B. Maury, Submitted, hal-03790427

Stable and convex contact algorithms.

- Friction
- Spherical particles
- Hugo Martin [LJLL, IPGP]



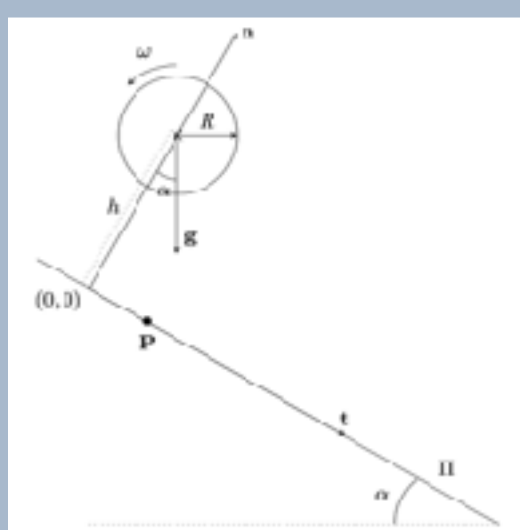
An optimization-based model for dry granular flows: application to granular collapse on erodible beds, with H. Martin, A. Mangeney, Y. Maday, B. Maury, Submitted, hal-03790427

Stable and convex contact algorithms.

- Convergence
- Non-spherical particles
- Hélène Bloch [CMAP]

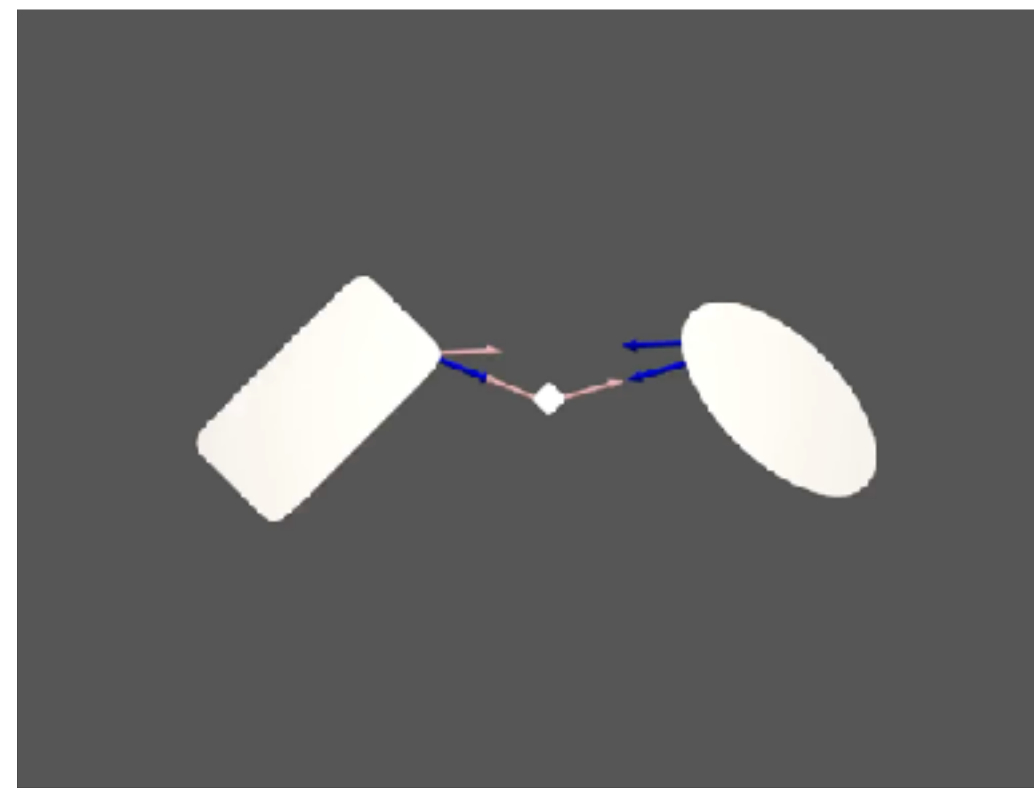
$$D^k + dt\nabla D^k \cdot \mathbf{v}^{k+1} \geq \mu\, dt\, |T\mathbf{u}^{k+1}|$$

- Influence of convexification?
- Very few convergence results based on compacity methods
- Order of convergence ?



Numerical study :

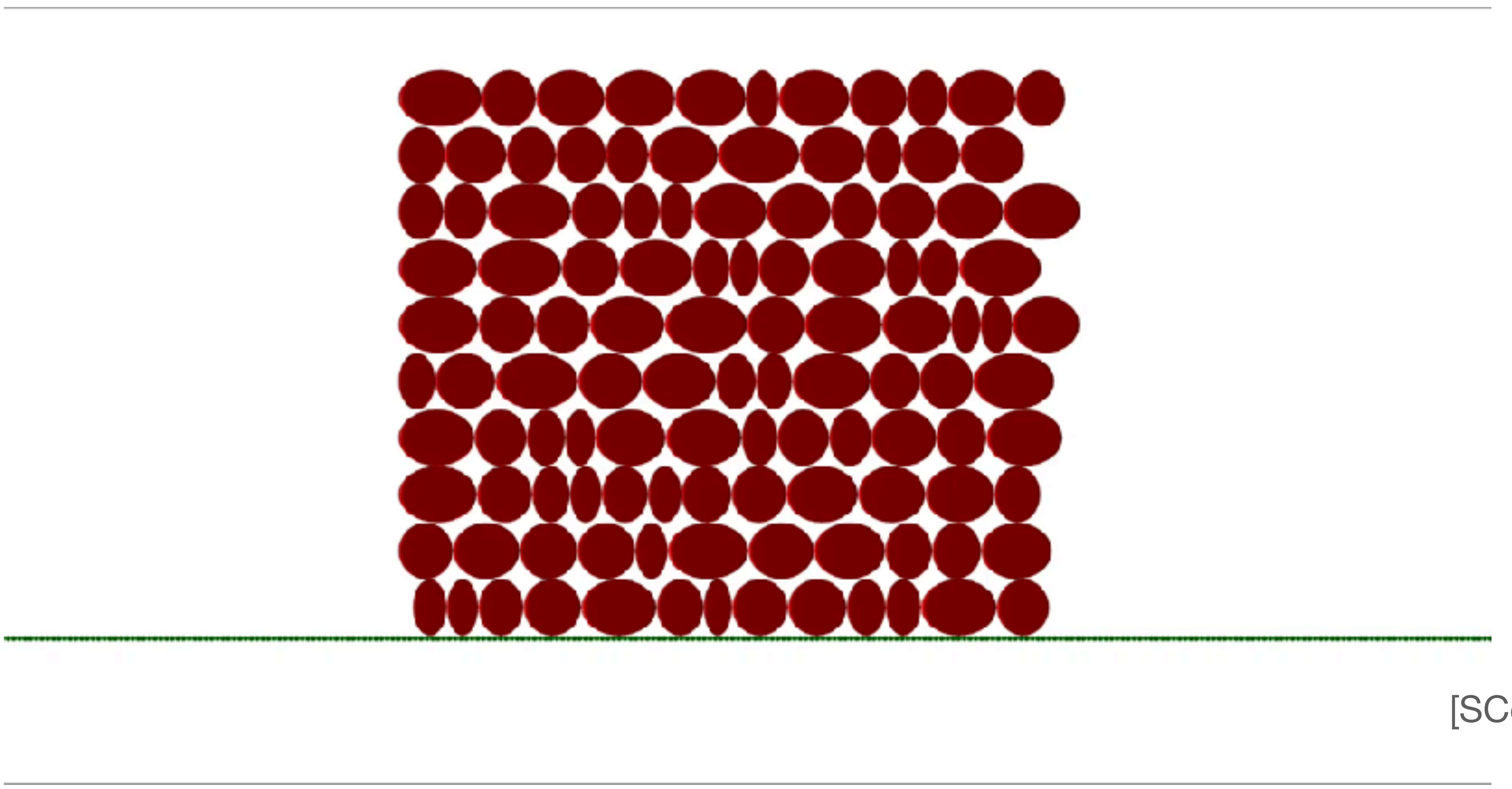
$$\mathrm{err} = O(dt)$$



On convex numerical schemes for inelastic contacts with friction, with H. Bloch, Proceeding SMAI2021, Submitted

Stable and convex contact algorithms.

- Convergence
- Non-spherical particles
- Hélène Bloch [CMAP]

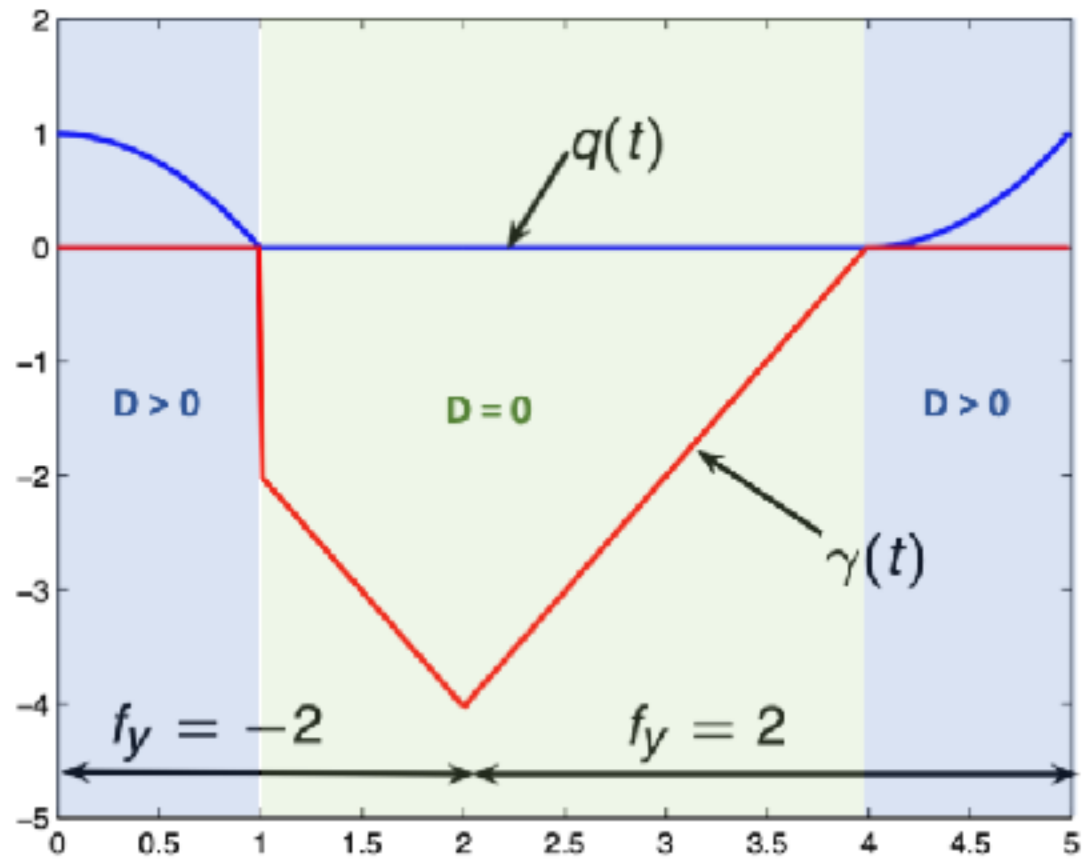


[SCoPI]

On convex numerical schemes for inelastic contacts with friction, with H. Bloch, Proceeding SMAI2021, Submitted

Stable and convex contact algorithms.

- Coupling with the gluey particle model

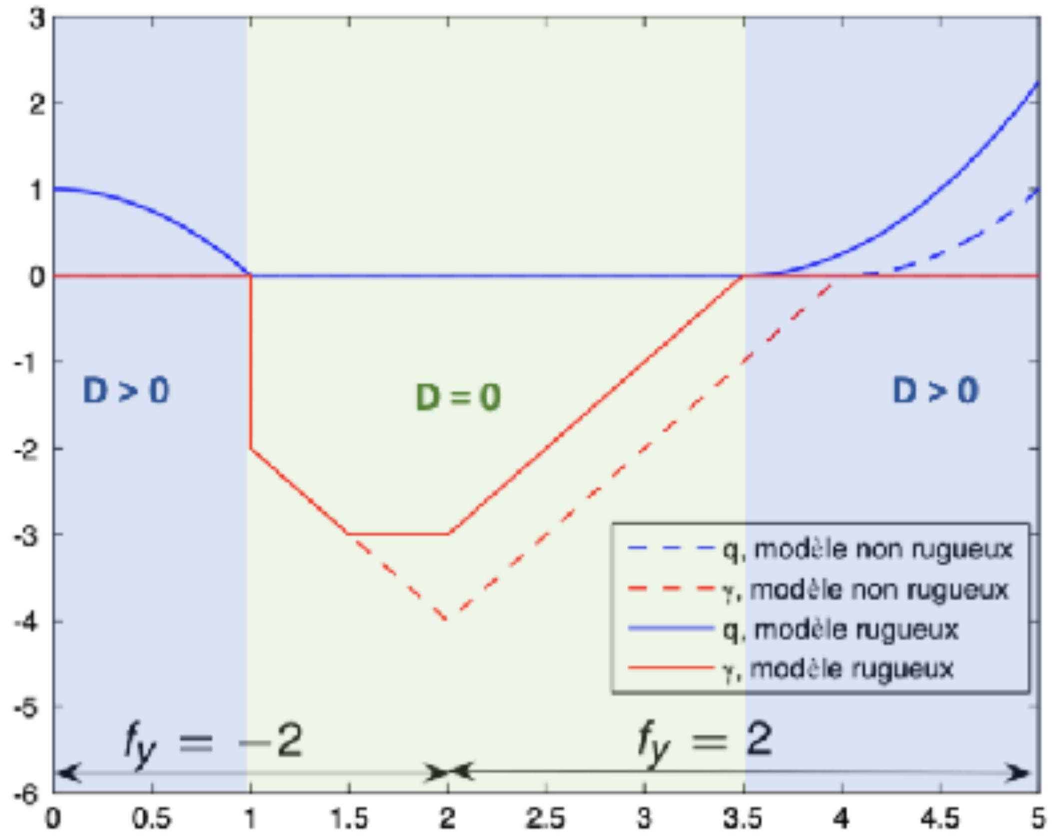


A gluey particle model, B. Maury, Esaim:Proc 18 (2007)

Numerical simulation of gluey particles, M2AN 43 (2009)

Stable and convex contact algorithms.

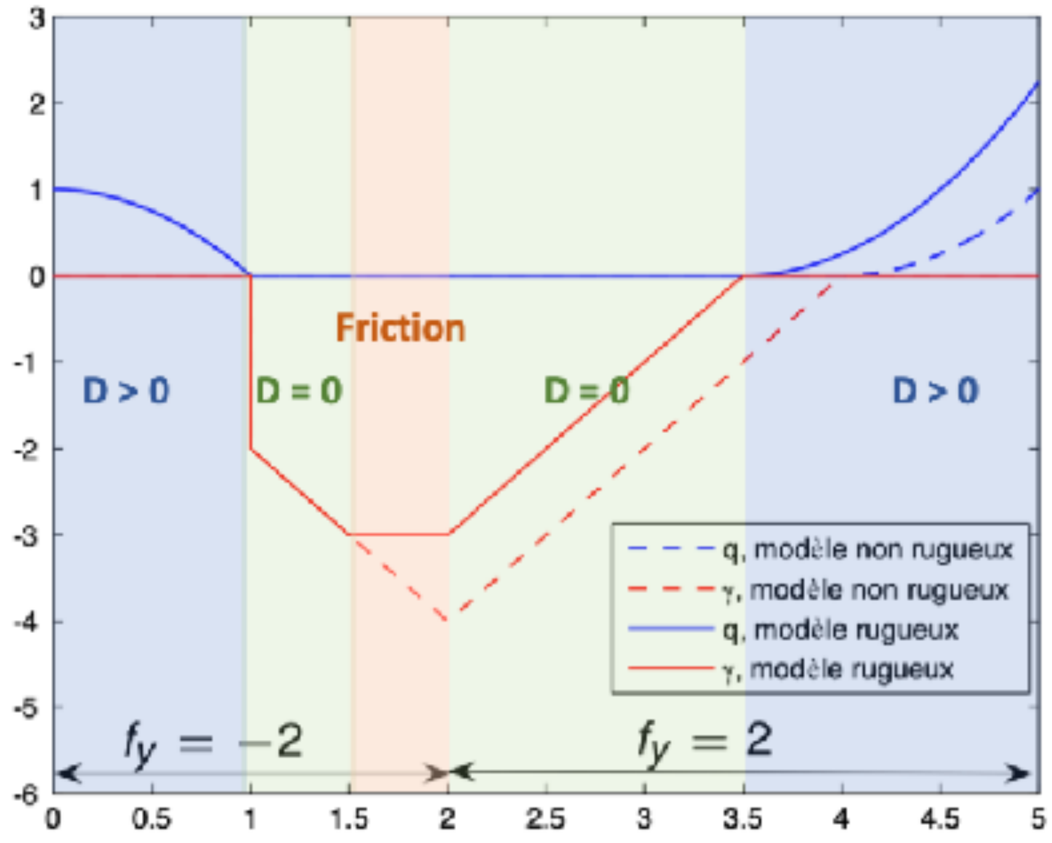
- Coupling with the gluey particle model



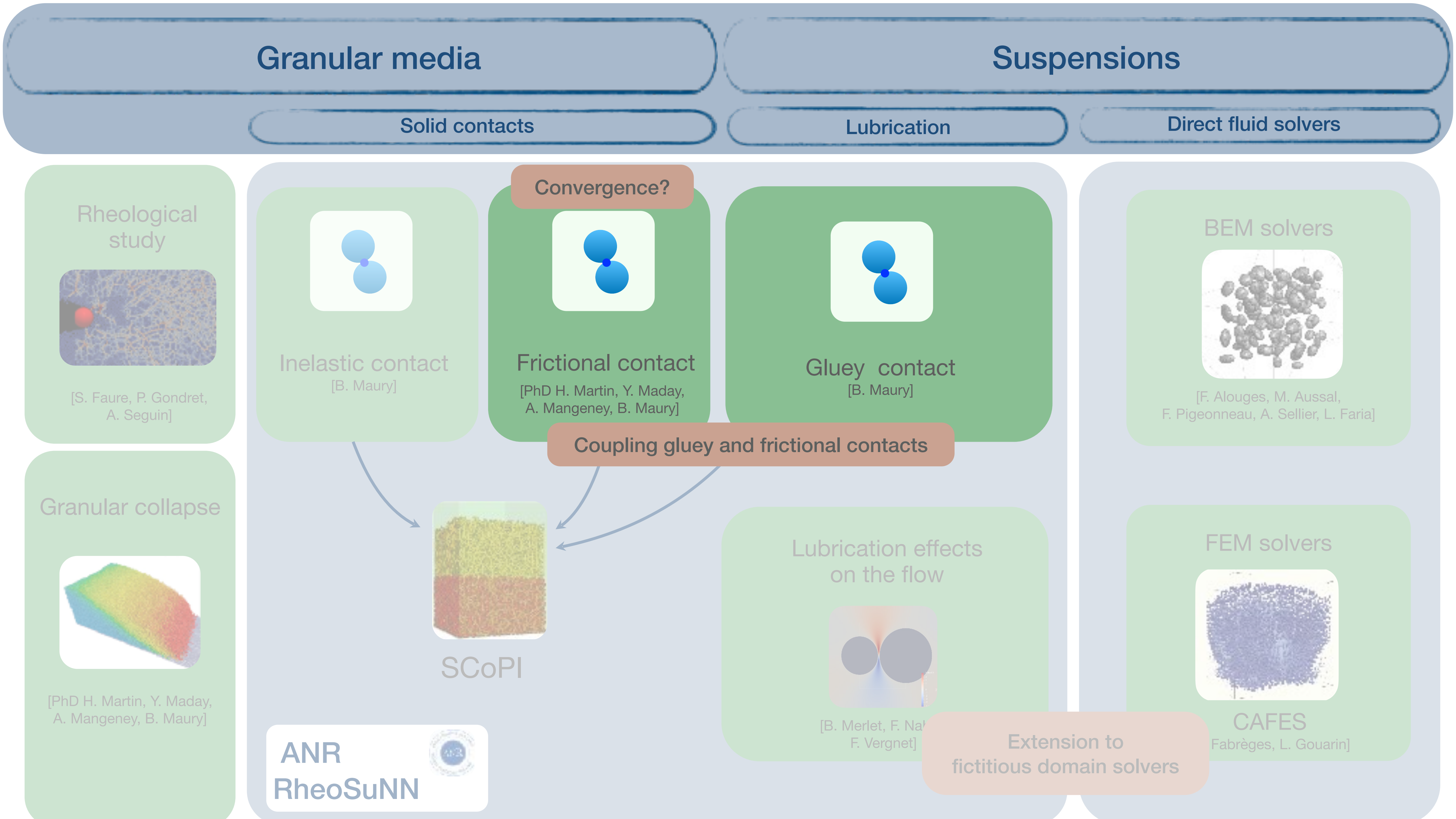
Numerical simulation of gluey particles, M2AN 43 (2009)

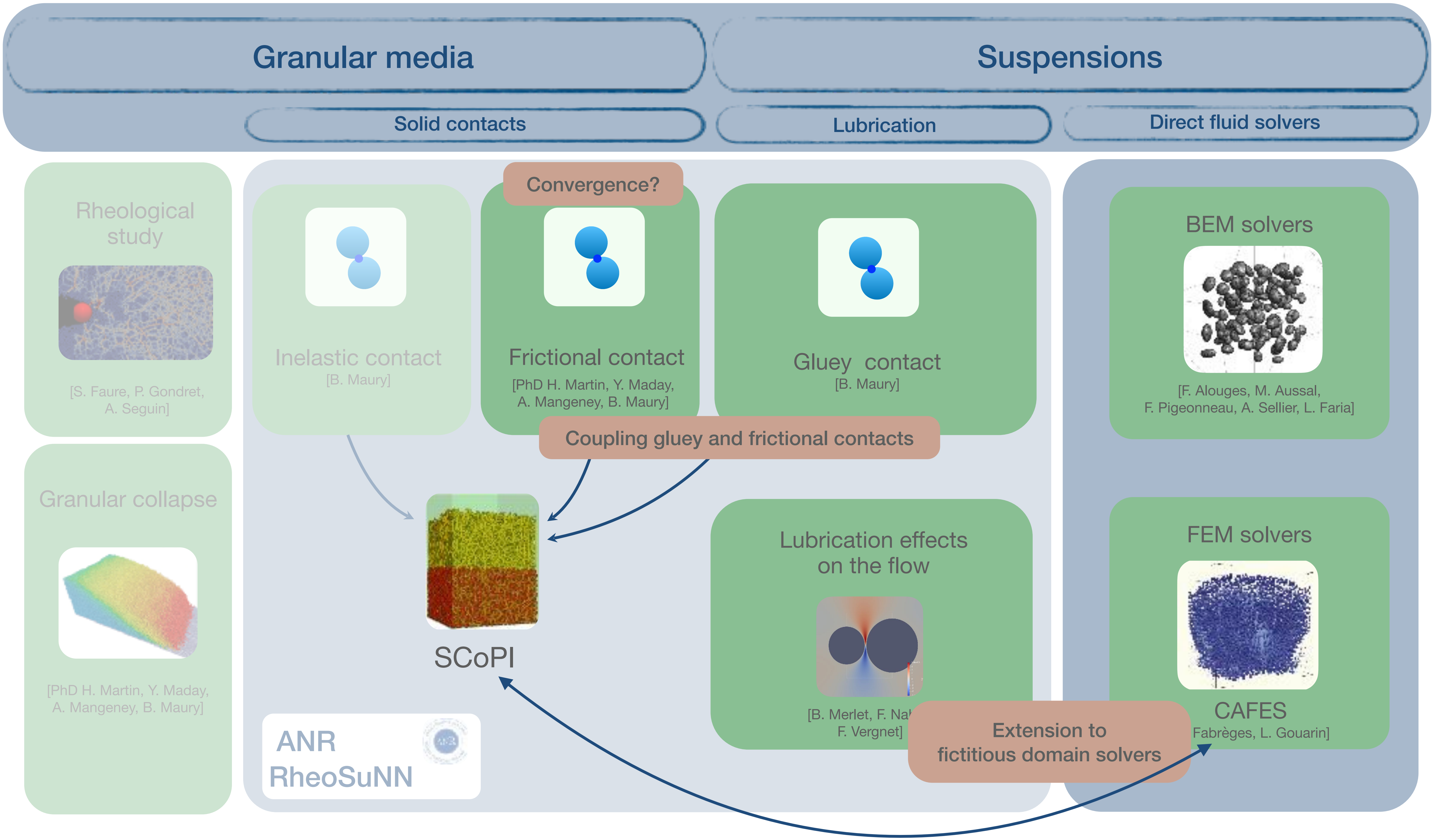
Stable and convex contact algorithms.

- Coupling with the gluey particle model



Stable and convex contact algorithms.





Granular media
Suspensions
Solid contacts
Lubrication
Direct fluid solvers
Rheological study
[S. Faure, P. Gondret, A. Seguin]
Granular collapse
[PhD H. Martin, Y. Maday, A. Mangeney, B. Maury]
Convergence?
Inelastic contact
[B. Maury]
Frictional contact
[PhD H. Martin, Y. Maday, A. Mangeney, B. Maury]
Gluey contact
[B. Maury]
Coupling gluey and frictional contacts
SCoPI
ANR RheoSuNN
Lubrication effects on the flow
[B. Merlet, F. Na
F. Vergnet]
Extension to fictitious domain solvers
BEM solvers
[F. Alouges, M. Aussal, F. Pigeonneau, A. Sellier, L. Faria]
FEM solvers
CAFES
Fabrèges, L. Gouarin]