

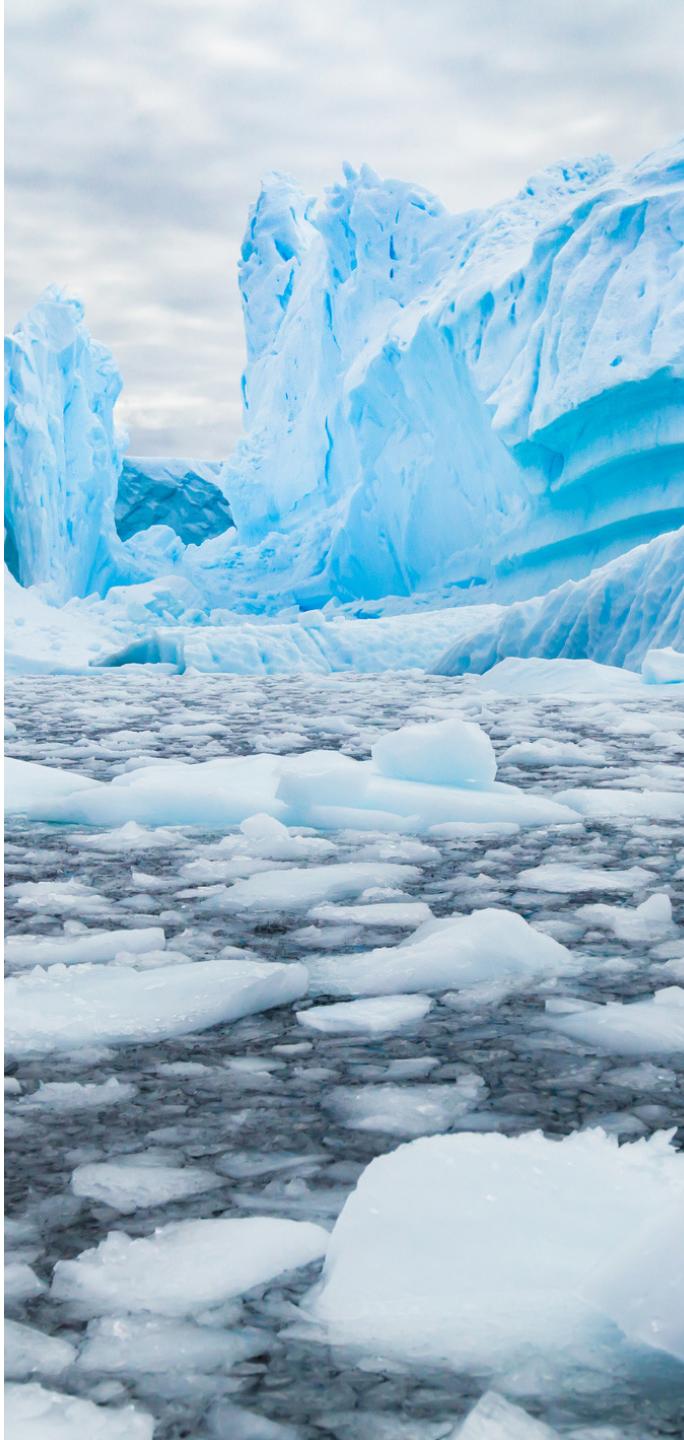
École 2022 du GdR CNRS MathGeoPhy
26 au 28 octobre 2022

Ice, glacier and ice-sheet flow

Olivier Gagliardini



*Tipping Points in Antarctic
Climate Components*



Who I am?

- 1993: ENS Cachan – *Agrégation* in Civil Engineering
- 1995-99: PhD on ice anisotropy at LGGE
- 1999: *Maître de conférences* UJF & LGGE
- 2010-2015 : *Institut Universitaire de France*
- 2012: *Professeur* University Grenoble Alpes

Modelling of complex ice flows

Interested in the **processes** that control glacier and ice-sheet dynamics

Who I am not?

- I am not a mathematician
- I am not even a numerical specialist

Who has contributed?

- **Colleagues** at IGE: Fabien Gillet-Chaulet, Gael Durand, Christian Vincent, Florent Gimbert, Adrien Gilbert,...

- **PhD students**

Fabien Gillet-Chaulet (CR, CNRS) 2003-2006

Basile de Fleurian (postdoc Norway) 2007-2010

Cyrille Mosbeux (postdoc IGE) 2013-2016

Julien Brondex (postdoc CEN) 2014-2017

Olivier Passalacqua (IGN) 2014-2017

Juan Pedro Roldan Blasco (ANR SAUSSURE) 2019-

Benoit Urruty (H2020 TiPACCs) 2019-

- **Post-docs**

Adrien Gilbert (CR CNRS) 2020-2022

Lionel Favier 2009-2012 ; 2018

Ma Ying 2009-2010

Gael Durand (DR CNRS) 2007-2009

Essentially with one tool



Contributor of the developments of the open source finite element code **Elmer/Ice**

A code dedicated to solve ice, glaciers and ice-sheet flow

Elmer/Ice is an add-on package to Elmer, a multi-physics FEM suite mainly developed by CSC-IT Center for Science Ltd. (Finland)

“Father” of the **Elmer/Ice** code

Animation of the community: more than 17 courses since 2018

193 publications using Elmer/Ice since 2004

<http://elmerice.elmerfem.org>

Elmer/Ice labeled “Code Communautaire” by INSU CNRS

Ice, glacier and ice-sheet flow

- Introduction: Cryosphere and climate change
- Ice(s), a material with a complex rheology
- Glaciers and Risks in a warming climate
- Grounding line and friction

Ice, glacier and ice-sheet flow

- **Introduction: Cryosphere and climate change**
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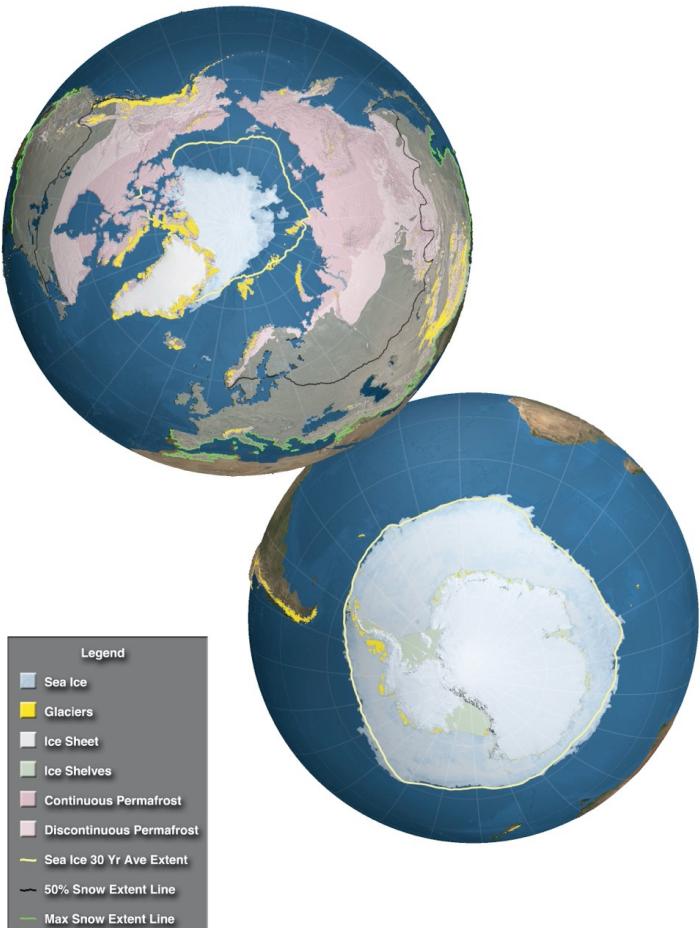
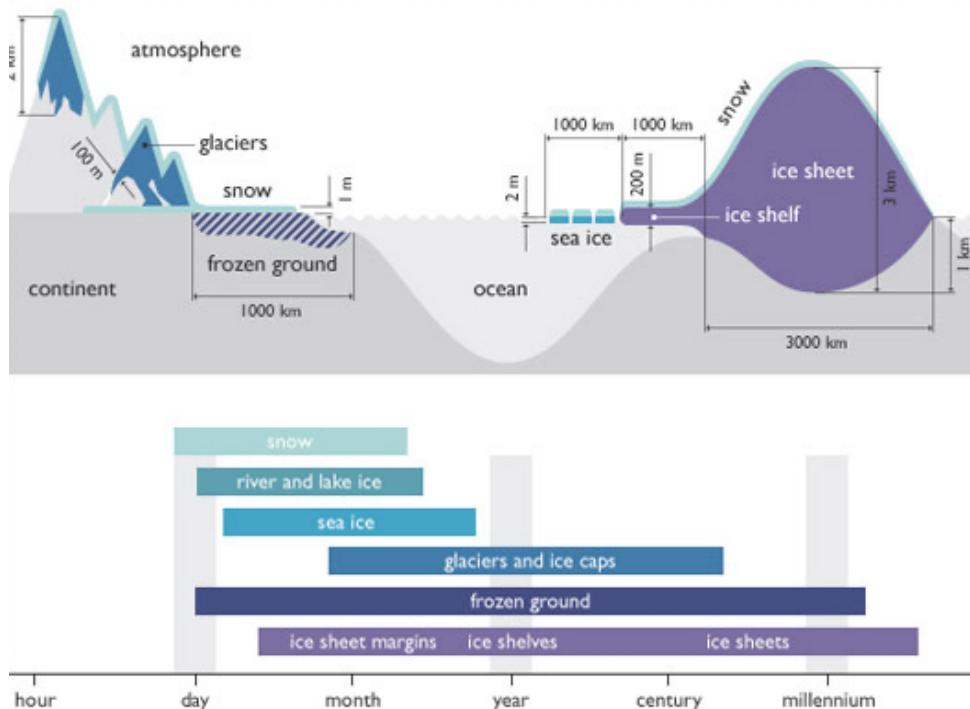
Cryosphere on Earth

Portions of Earth's surface where water is in solid form

Includes: sea ice, lake ice, river ice, snow cover, **glaciers**, **ice caps**, **ice sheets**, and frozen ground (incl. permafrost)

Plays a significant role in the global climate

Response times vary from days to milleniums

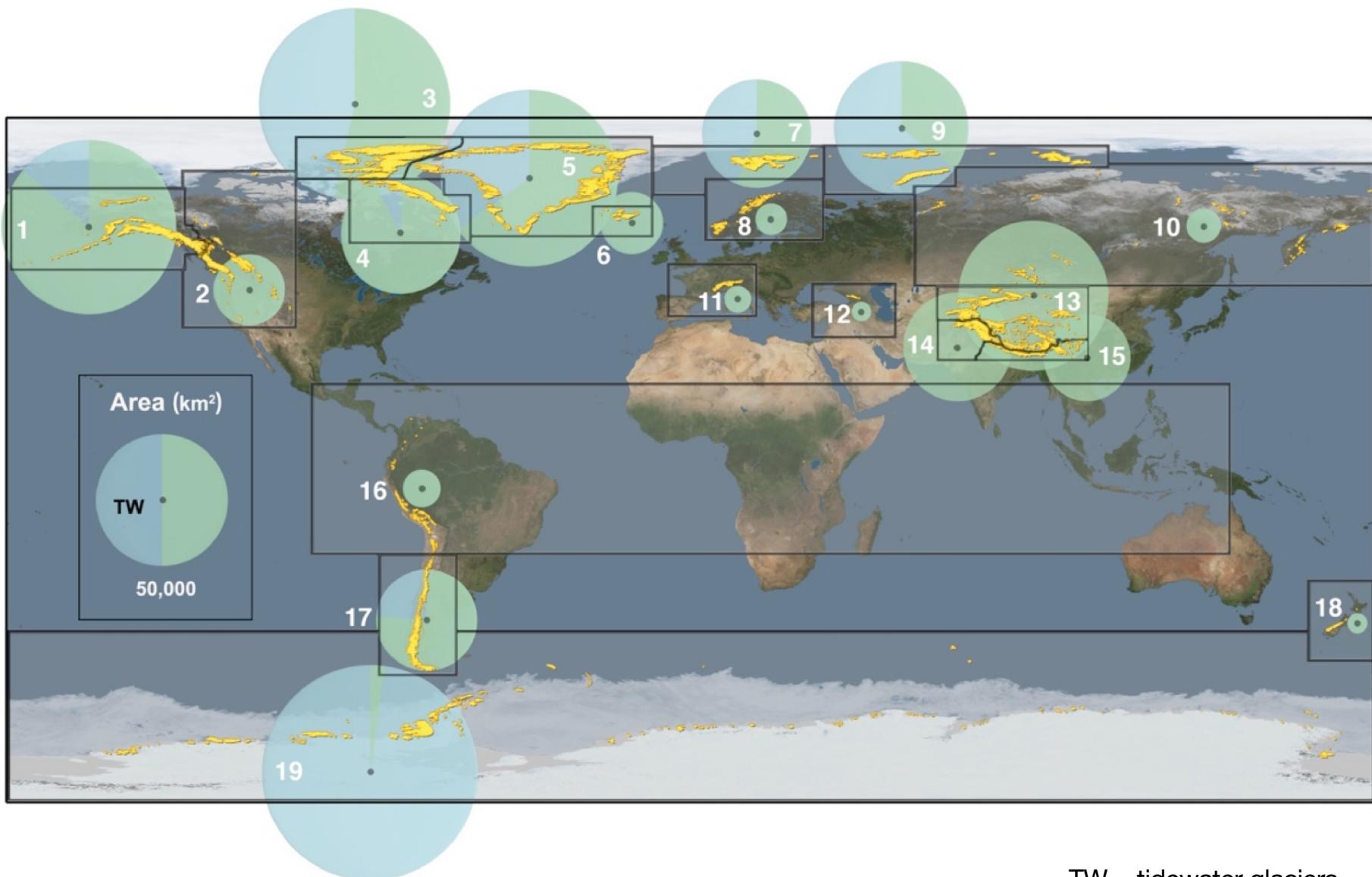


Areas and Volumes

Ice on Land	Percent of Global Land Surface ^a	Sea Level Equivalent ^b (metres)
Antarctic ice sheet ^c	8.3	58.3
Greenland ice sheet ^d	1.2	7.36
Glaciers ^e	0.5	0.41
Terrestrial permafrost ^f	9–12	0.02–0.10 ^g
Seasonally frozen ground ^h	33	Not applicable
Seasonal snow cover (seasonally variable) ⁱ	1.3–30.6	0.001–0.01
Northern Hemisphere freshwater (lake and river) ice ^j	1.1	Not applicable
Total^k	52.0–55.0%	~66.1
Ice in the Ocean	Percent of Global Ocean Area ^a	Volume ^l (10 ³ km ³)
Antarctic ice shelves	0.45 ^m	~380
Antarctic sea ice, austral summer (spring) ⁿ	0.8 (5.2)	3.4 (11.1)
Arctic sea ice, boreal autumn (winter/spring) ⁿ	1.7 (3.9)	13.0 (16.5)
Sub-sea permafrost ^o	~0.8	Not available
Total^p	5.3–7.3	

1 mm SLE ~ 362.5 Gt of ice

Glaciers and Ice caps: repartition of Earth



Glaciers and Ice caps: definitions

Glaciers:

- Small land-based ice masses in mountainous regions
- Constrained by topographical features
- Are found on every continent
- Many different types: Valley glaciers, cirque glaciers, hanging glaciers, tidewater glaciers
- Important fresh water resource for millions of people



Ice caps and Icefields:

- Ice masses that cover less than 50,000 km² of land area
- Icefields are constrained by topographical features
- Ice caps override the underlying topography



Vatnajökull, Iceland



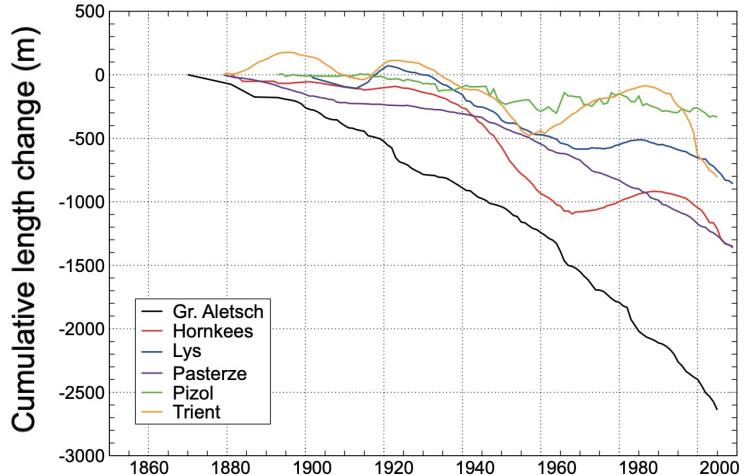
Southern Patagonian Ice Field

Glaciers length

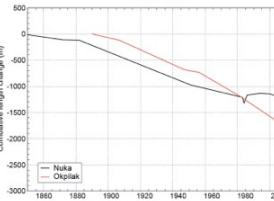
Worldwide retreat since the mid 19th century

Increased mass loss during the last decades

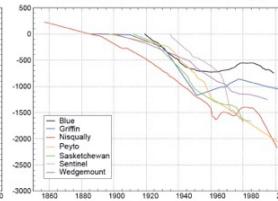
11 Central Europe



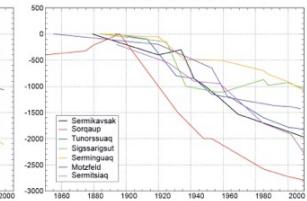
1 Alaska



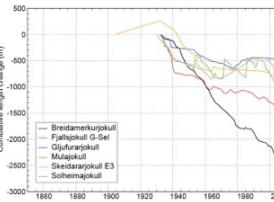
2 Western Canada and US



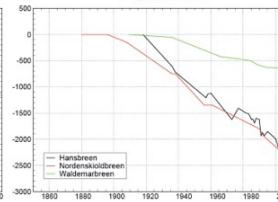
5 Greenland



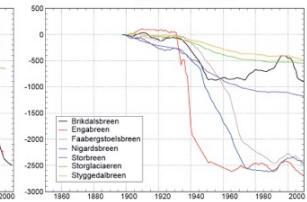
6 Iceland



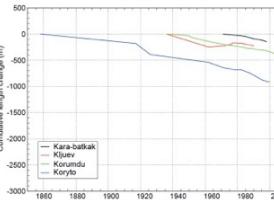
7 Svalbard



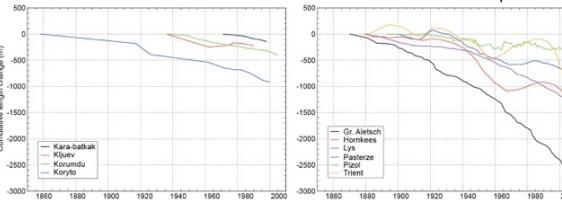
8 Scandinavia



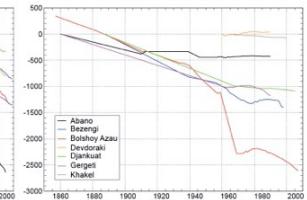
10 North Asia



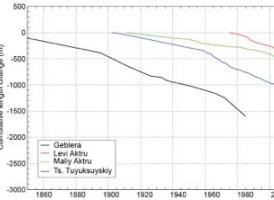
11 Central Europe



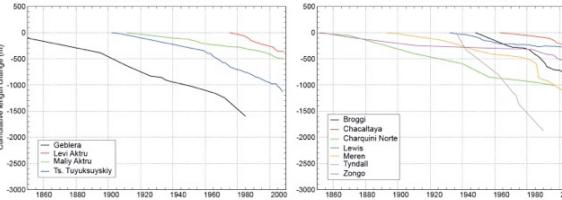
12 Caucasus and Middle East



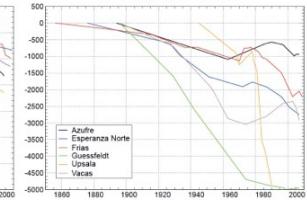
13 Central Asia



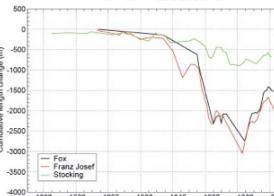
16 Low Latitudes



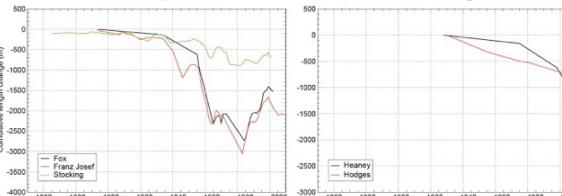
17 Southern Andes



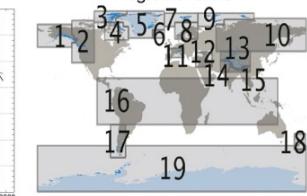
18 New Zealand



19 Antarctic and Subantarctic



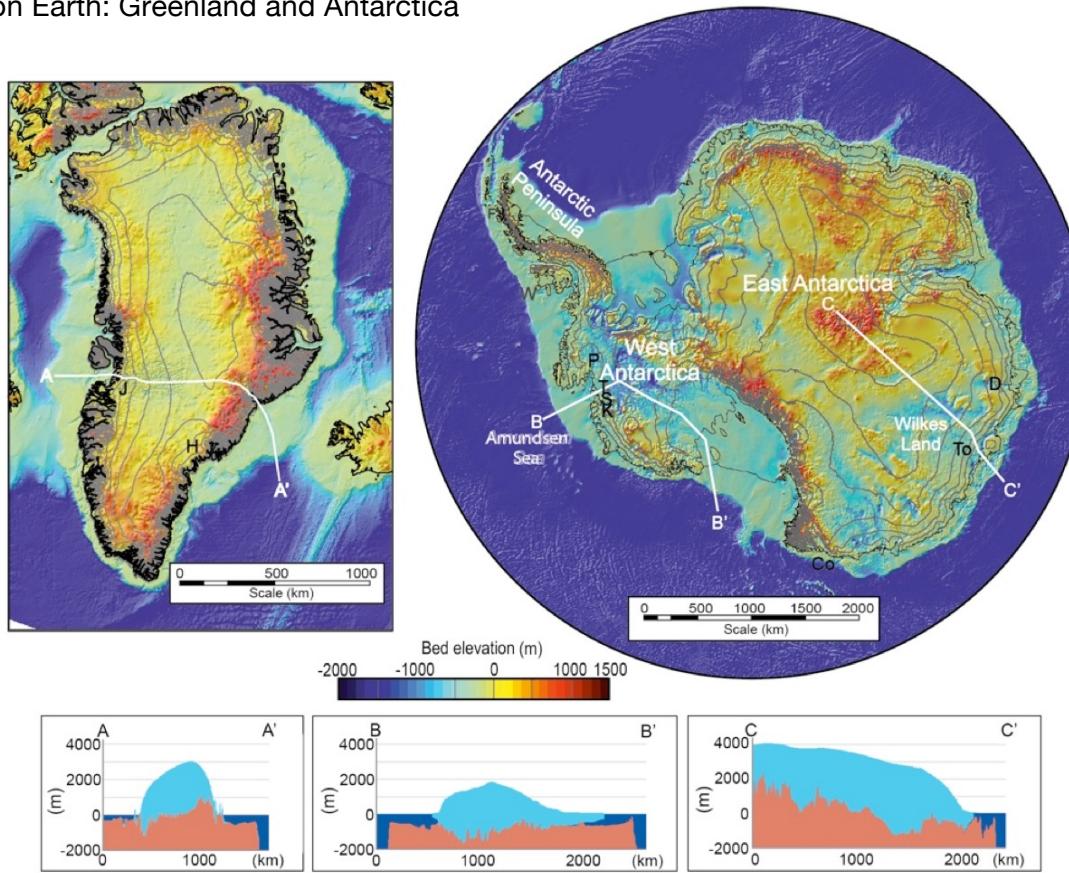
Region overview



Ice sheets

Ice masses that cover more than 50,000 km² of land area

Currently 2 ice sheets on Earth: Greenland and Antarctica



Glossary:

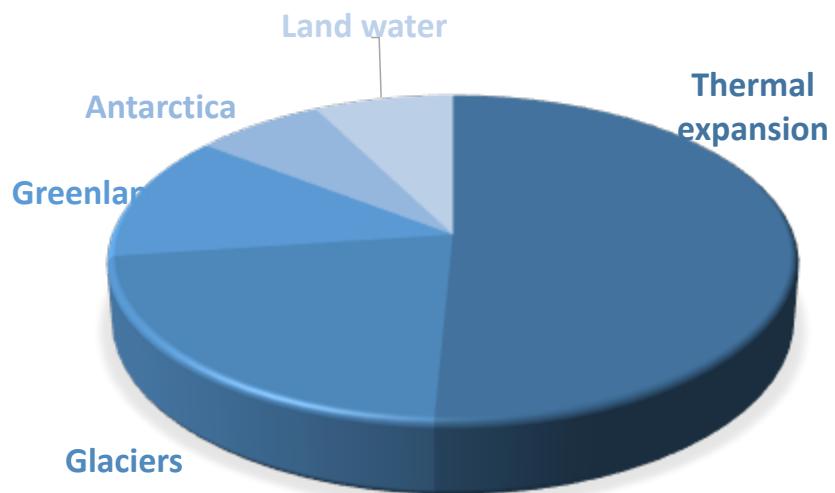
Ice shelf: A floating slab of ice of considerable thickness extending from the coast often filling embayments in the coastline of an ice sheet

Marine-based ice sheet: An ice sheet containing a substantial region that rests on a bed lying below sea level and whose perimeter is in contact with the ocean.

Increasing mass loss contributing to sea level rise

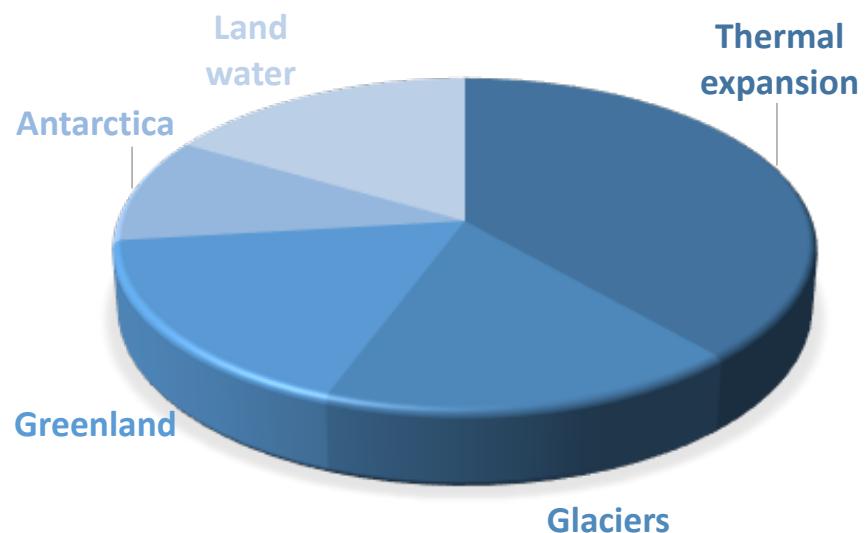
Components of sea level rise

1971-2018



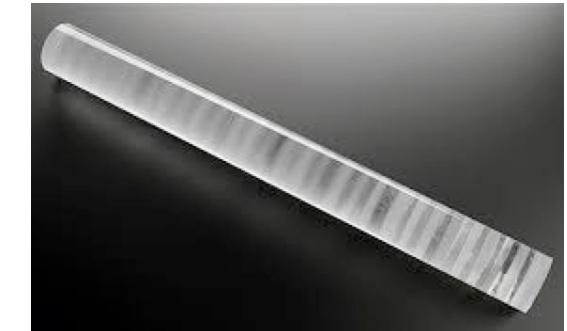
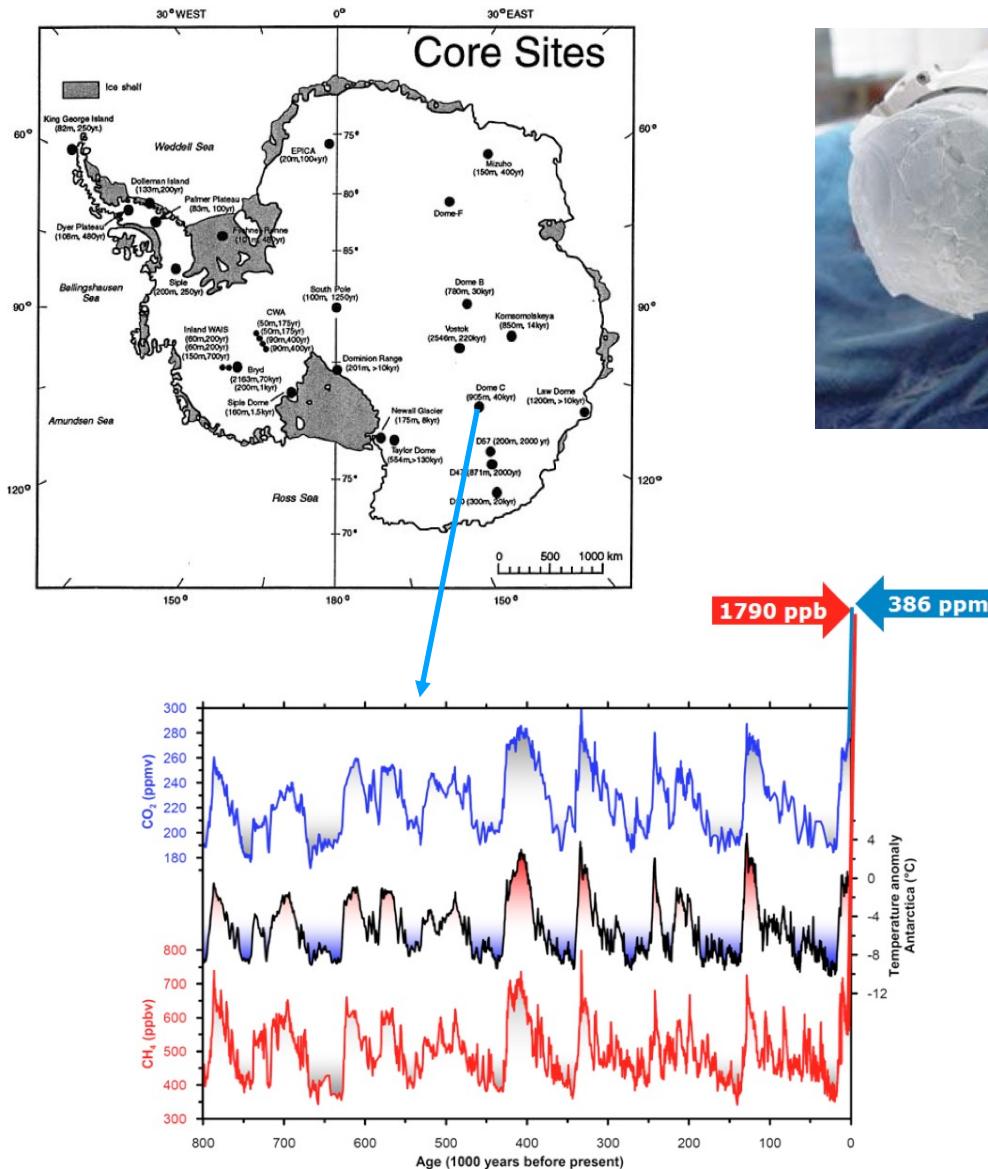
2.3 mm/year

2006-2018

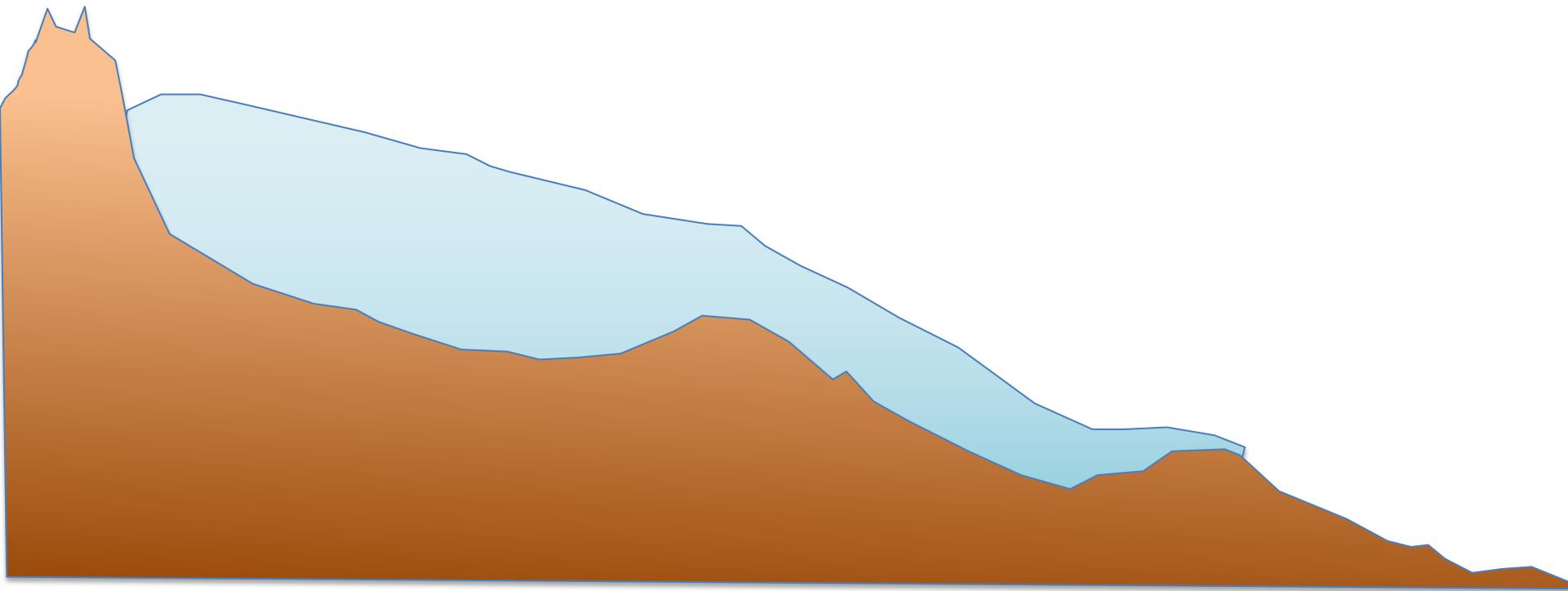


3.7 mm/year

Ice core and climate archives

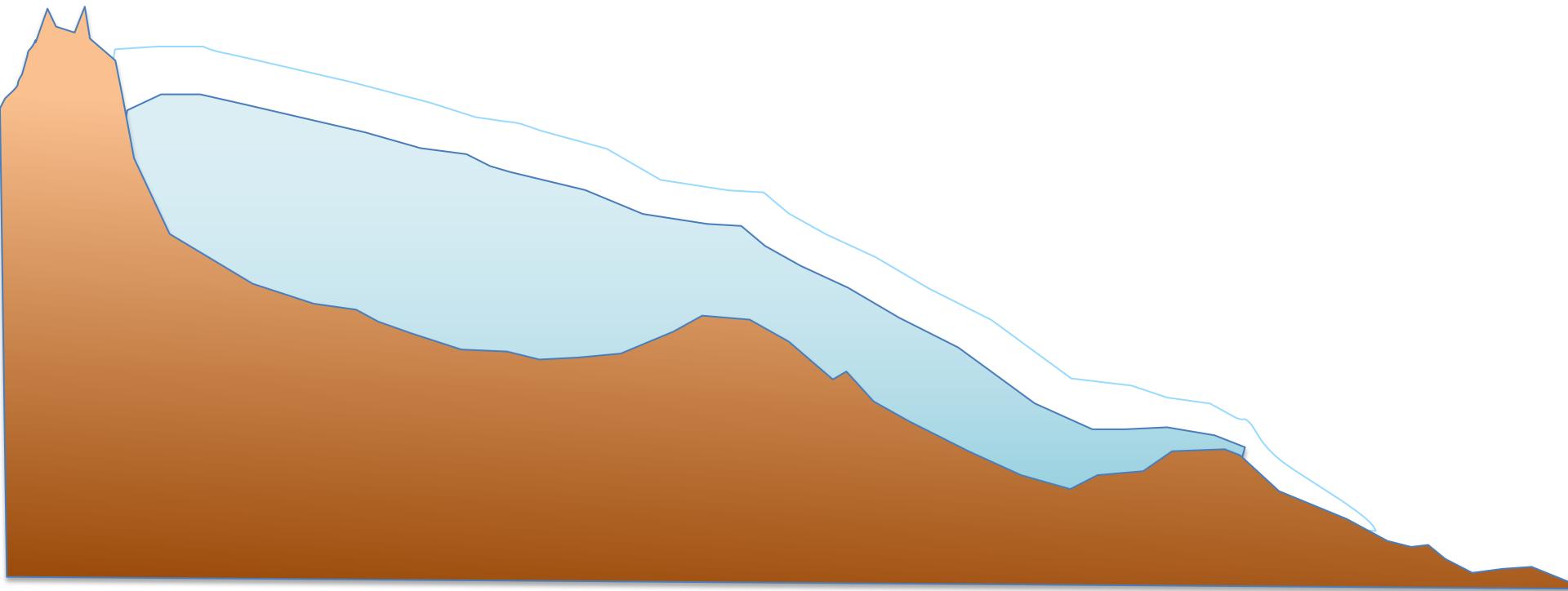


How does it work?



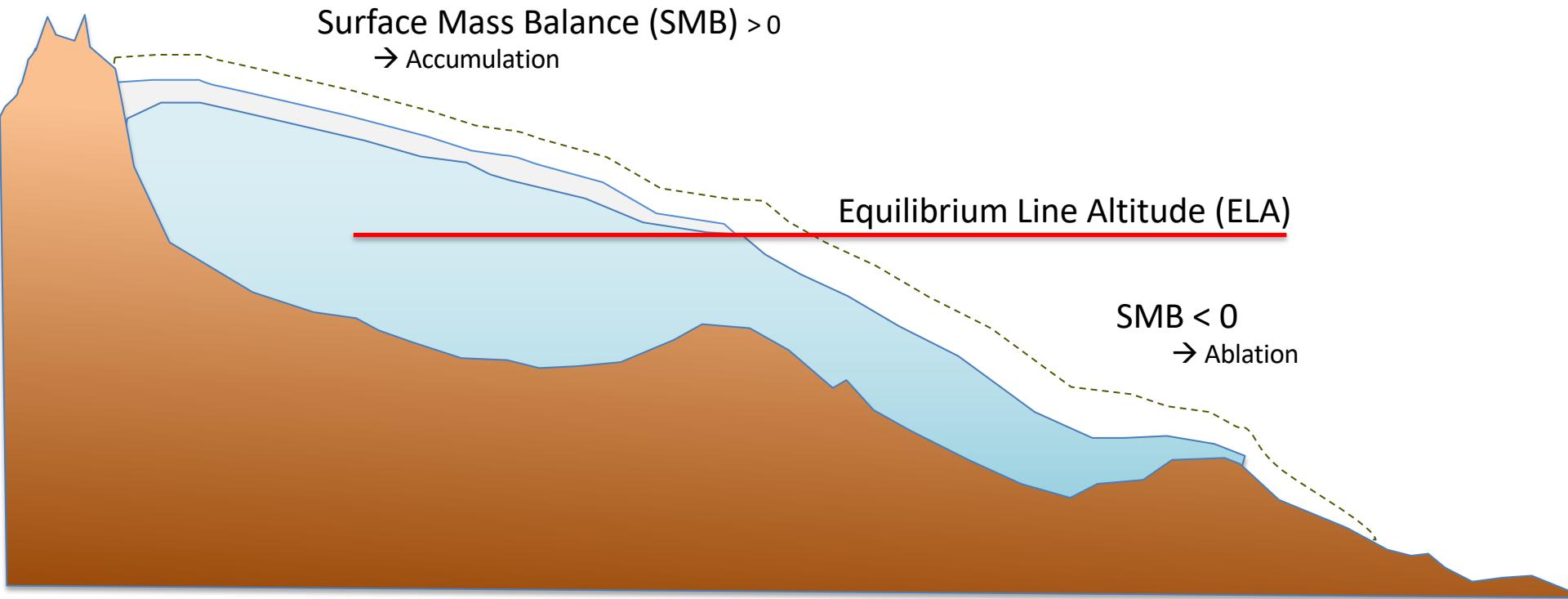
How does it work?

Snow accumulates during winter...

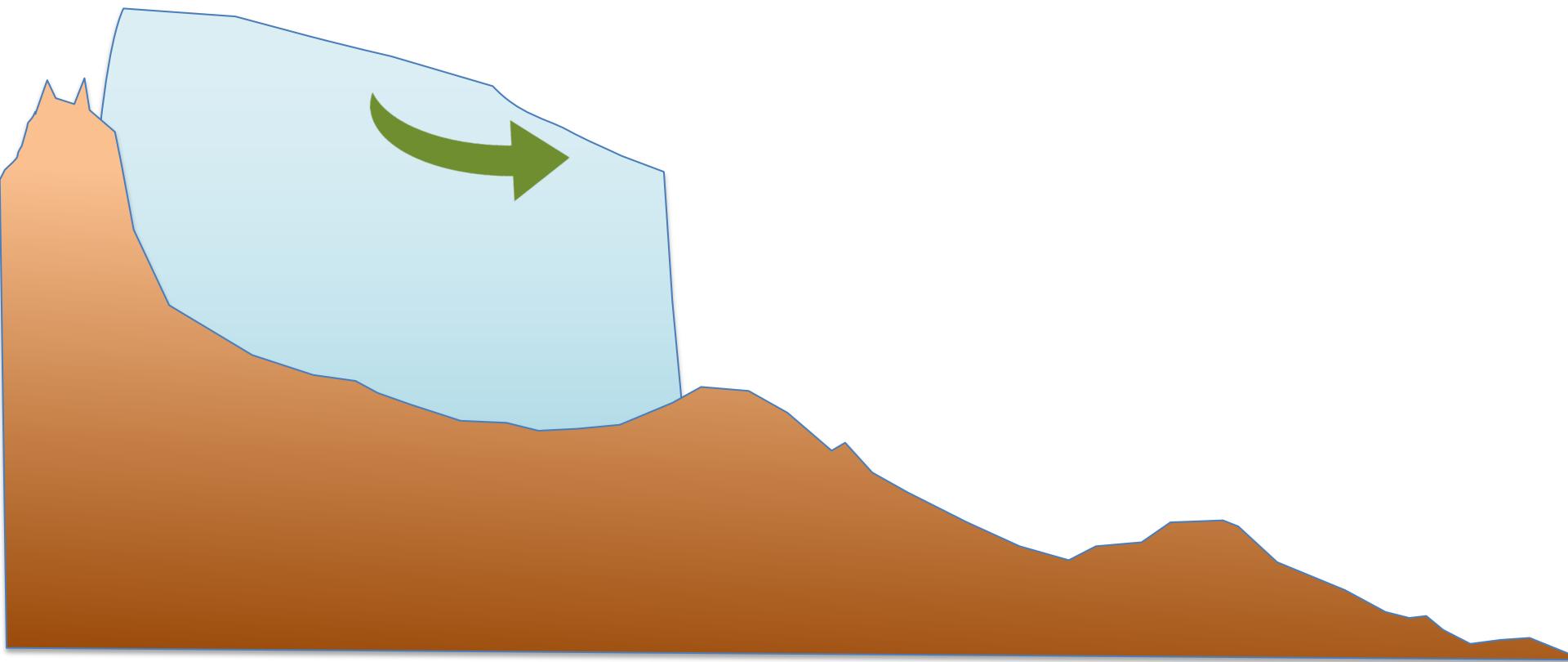


How does it work?

and melt partially during summer...

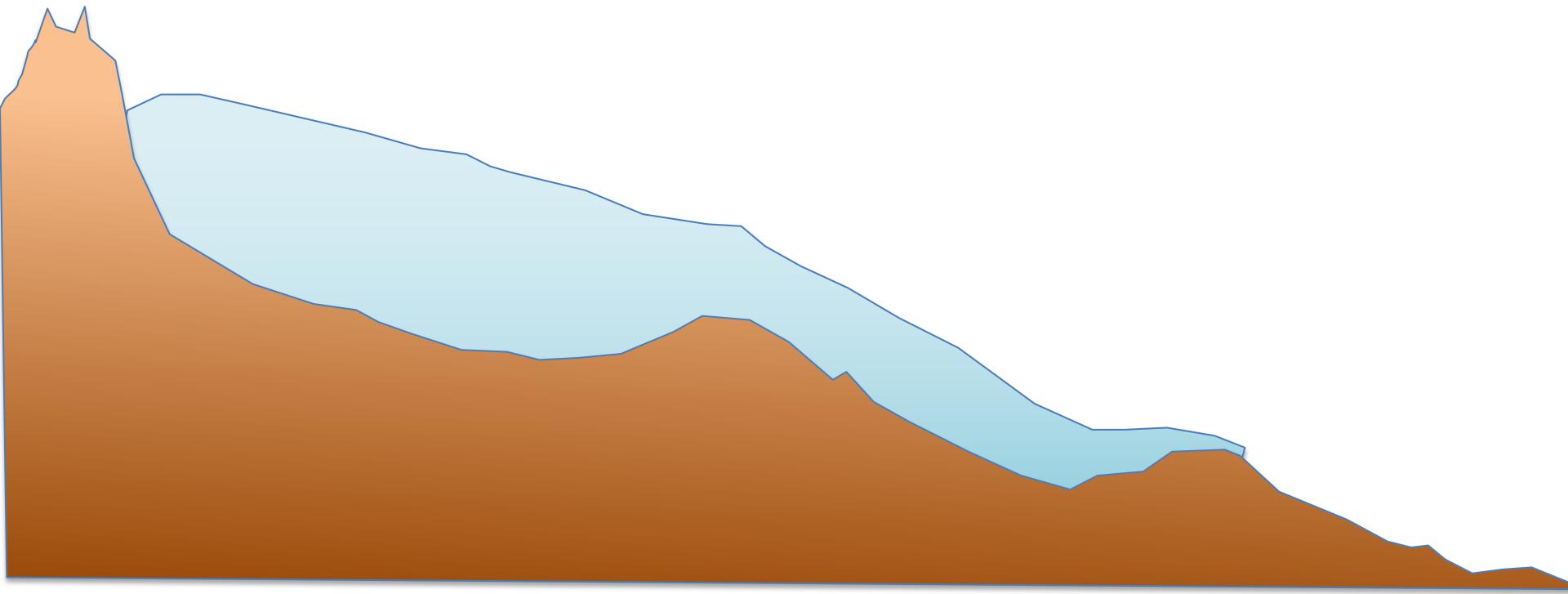


How does it work?



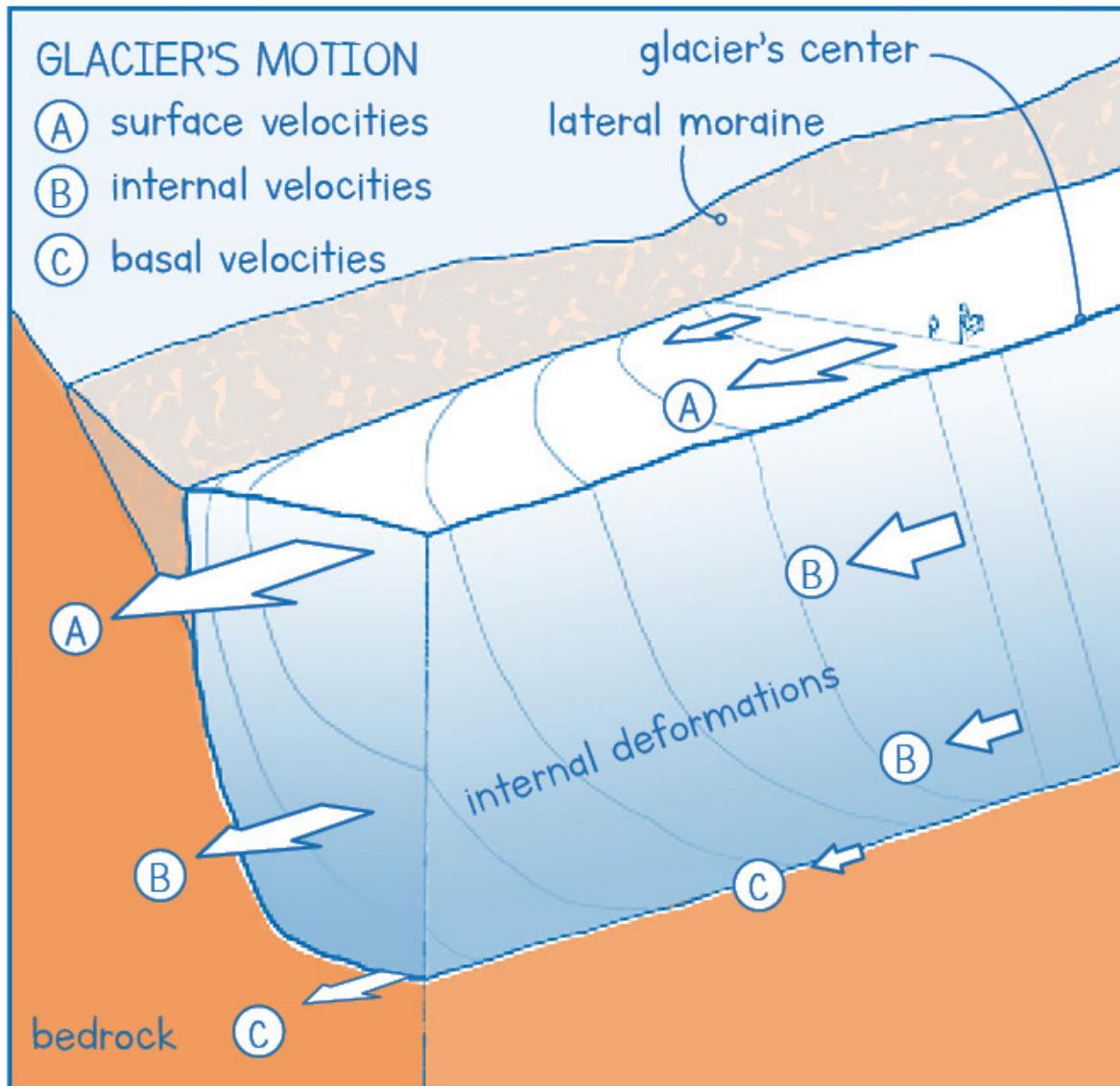
How does it work?

Glacier movements redistribute ice from top to bottom

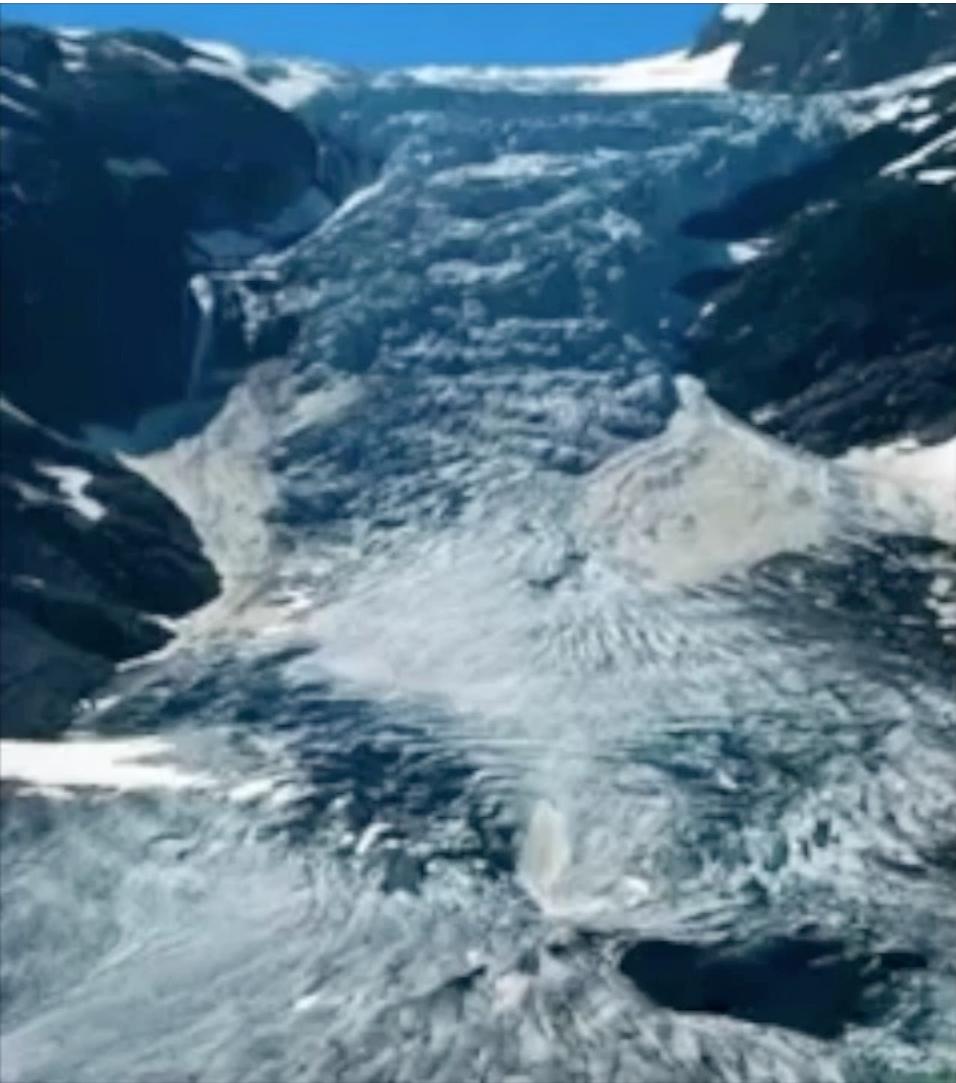


Glacier motion = internal deformation + basal sliding

Glacier motion = internal deformation + sliding



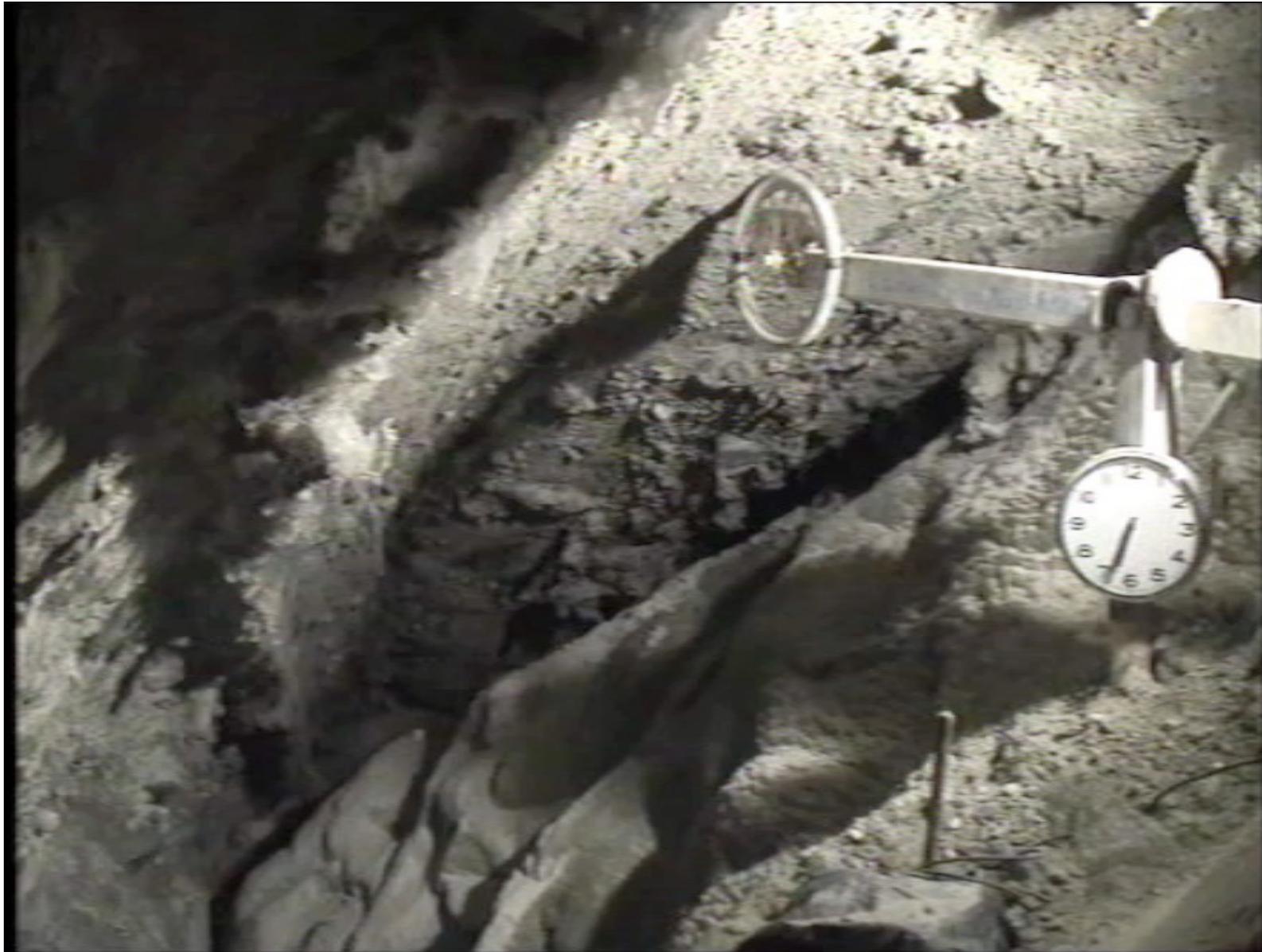
Non-linear viscous flow



Glace
=
Fluide visqueux

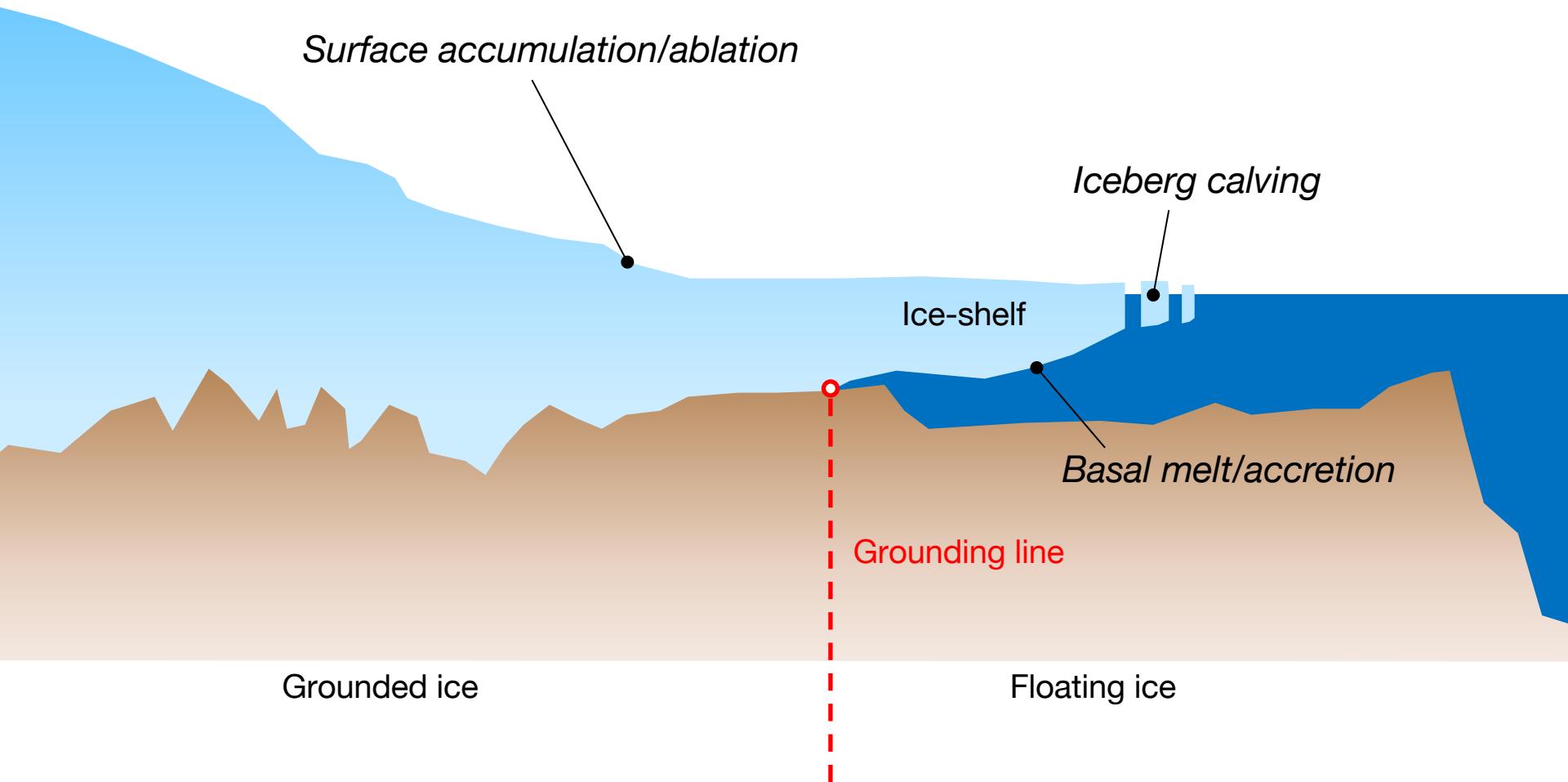
<https://www.geo.uzh.ch/~gjouvet/>

Basal sliding of ice over the bedrock



<http://www.moreauluc.com>

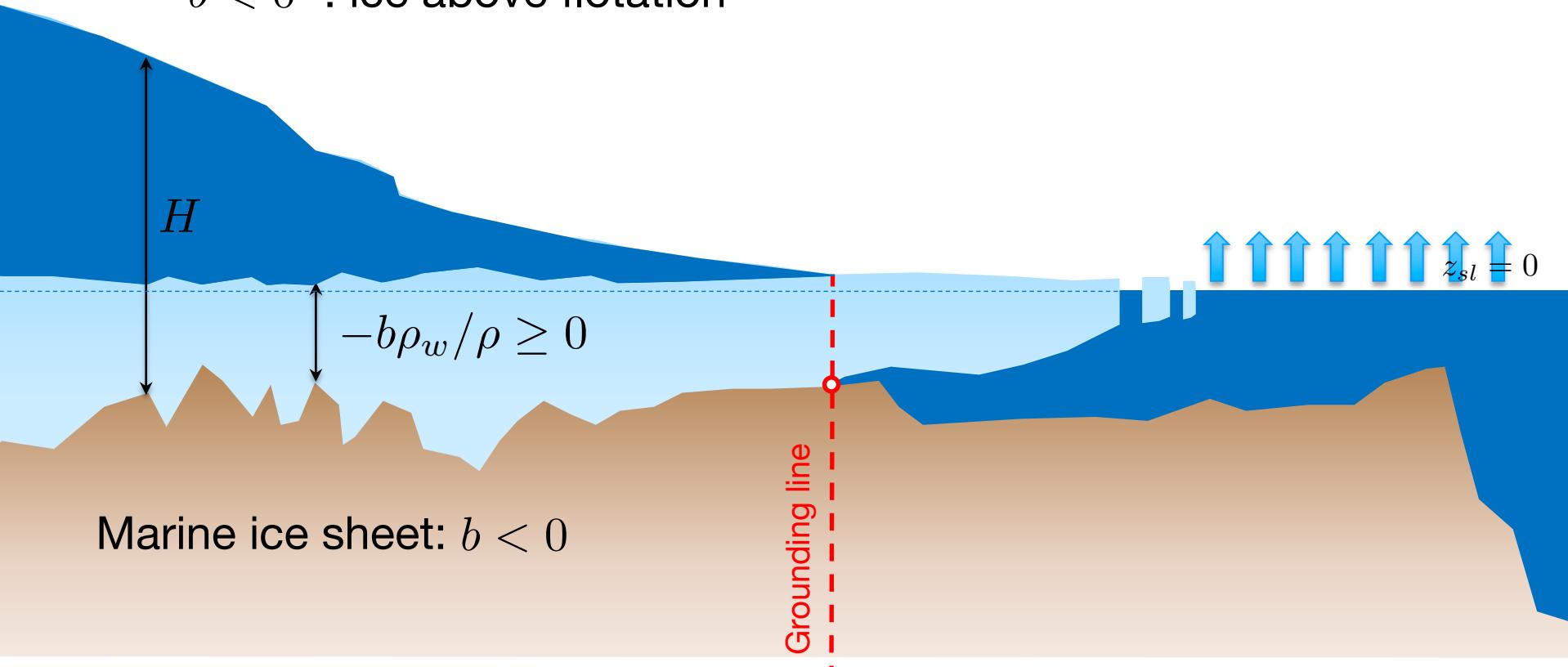
When ice meets water... other processes to loose/gain mass



Which ice contributes to Sea Level Rise (SLR)?

$b \geq 0$: all melted ice

$b < 0$: ice above flotation



Marine ice sheet: $b < 0$

Grounding line

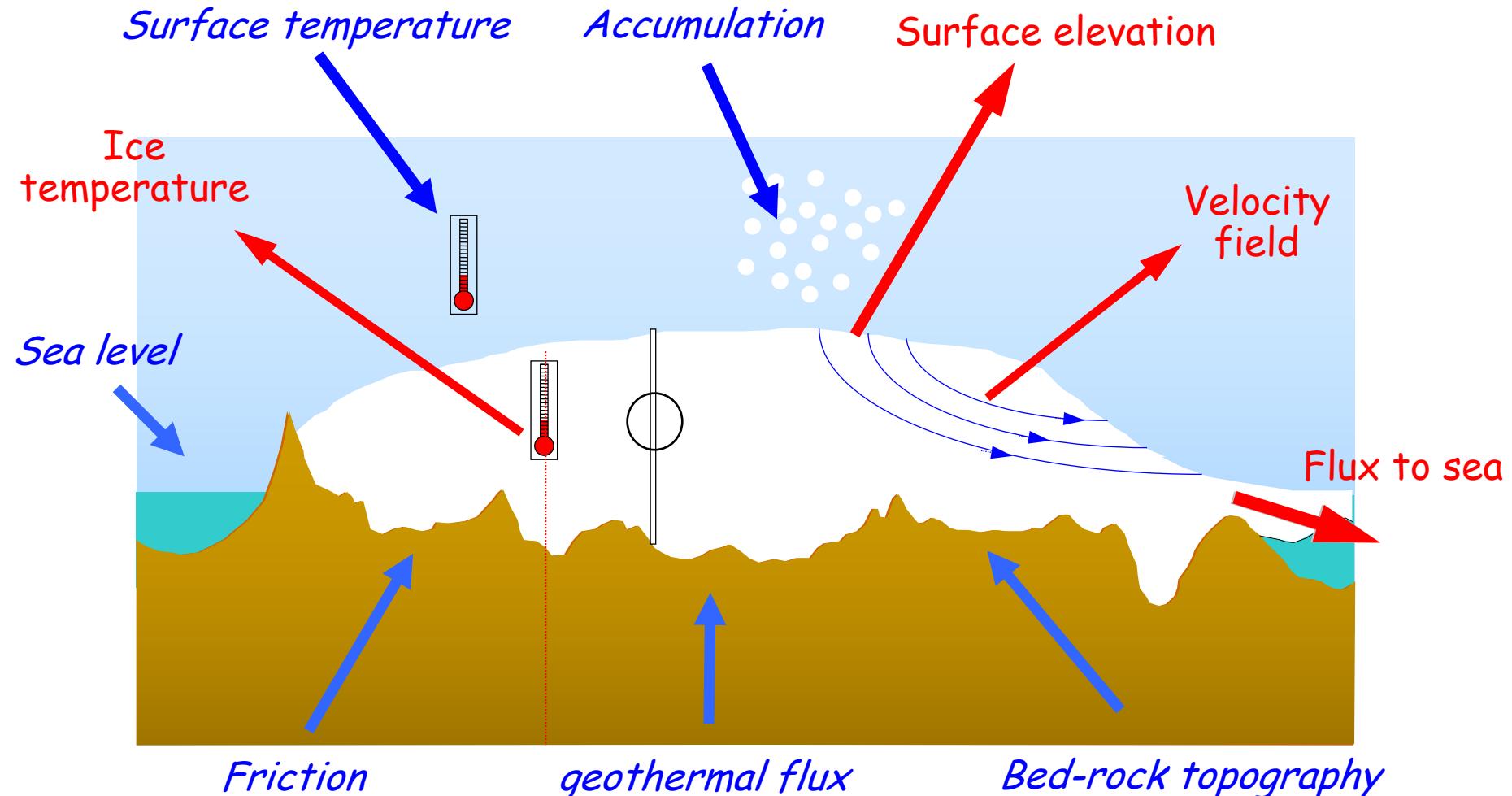
Only the ice above flotation is contributing to rise sea level!

Volume of Ice above flotation (VAF) → Sea Level Rise

Volume of water = Volume of ice x ice density (~0.91)

1 mm SLR = $362.5 \times 10^6 \text{ km}^2 \times 1\text{mm} = 362.5 \text{ km}^3 = 362.5 \text{ Gt of water}$

Problematic of glacier and ice sheet flow modelling



+ other eventual internal variables (density, fabric, damage,...)

Input

Output

Temperature: complexity of boundary conditions

Atmosphere/Ice interface:
- Surface temperature



Ocean/Ice interface:
- Sea temperature



Heat equation

Bedrock/Ice interface:
- Geothermal heat-flux
- Temperate (amount of melting) or Cold base



Ice flow: complexity of boundary conditions

Atmosphere/Ice interface:

- Free surface with accumulation/ablation

Ocean/Ice interface:

- Free surface with accretion/melting
- Calving of icebergs
- Sea level

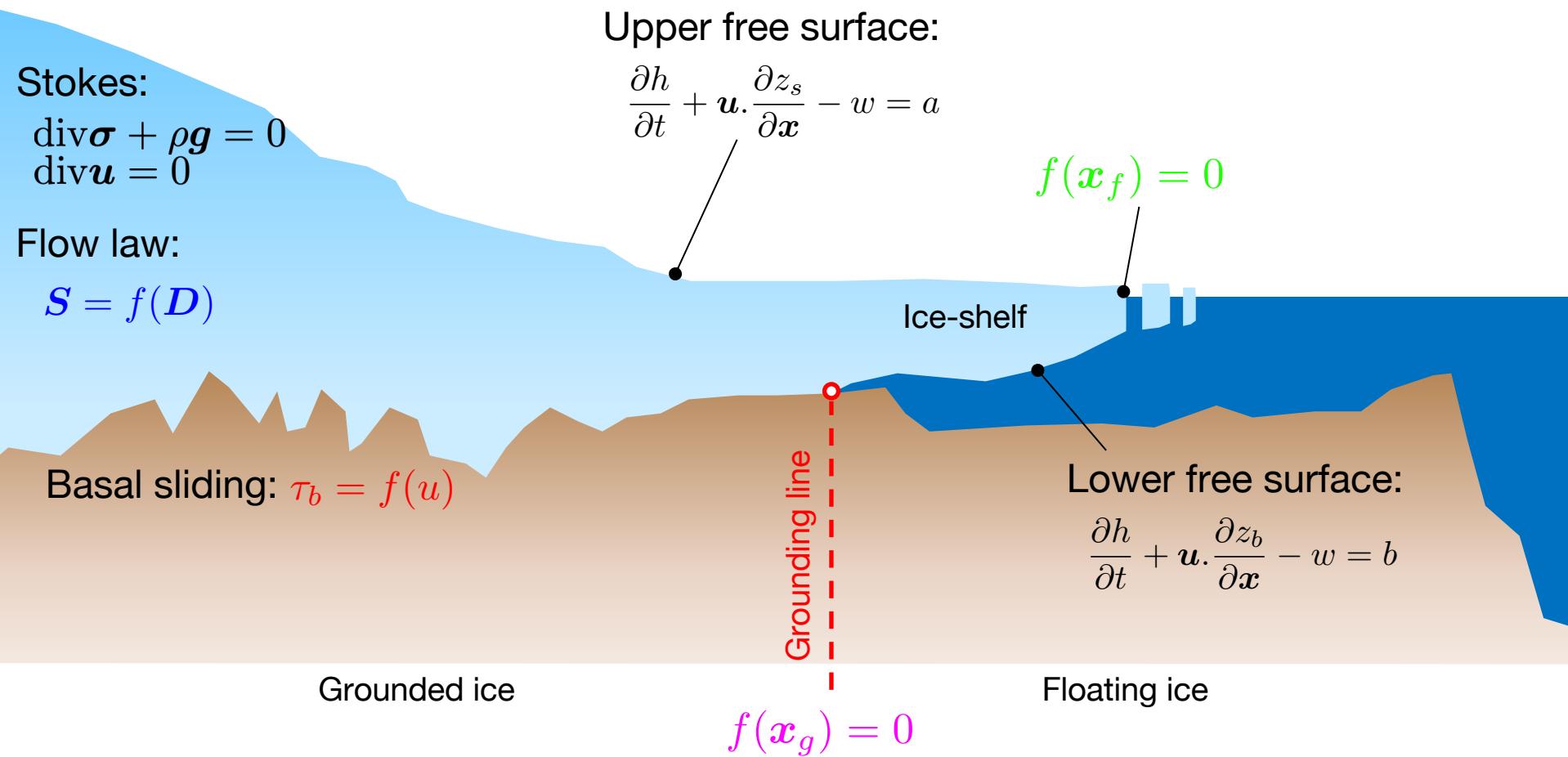
Ice flow

Bedrock/Ice interface:

- Friction function of roughness, water pressure, nature of rocks, ...
- Isostasy adjustment

And most boundaries are evolving surfaces (GL, front)

Which equations? Which boundary conditions?



- Ice, glacier and ice-sheet flow

- Introduction: Cryosphere and climate change
- **Ice(s), a material with a complex rheology**
- Glaciers and Risks in a warming climate
- Grounding line and friction

Which equations? Which boundary conditions?

Stokes:

$$\operatorname{div} \boldsymbol{\sigma} + \rho \mathbf{g} = 0$$
$$\operatorname{div} \mathbf{u} = 0$$

Flow law:

$$\mathbf{S} = f(\mathbf{D})$$

Basal sliding: $\tau_b = f(u)$

Grounded ice

Upper free surface:

$$\frac{\partial h}{\partial t} + \mathbf{u} \cdot \frac{\partial z_s}{\partial x} - w = a$$

$$f(x_f) = 0$$

Ice-shelf

Grounding line

$$f(x_g) = 0$$

Lower free surface:

$$\frac{\partial h}{\partial t} + \mathbf{u} \cdot \frac{\partial z_b}{\partial x} - w = b$$

Floating ice

Rheology of Ice(s)

✓ The Physics

- Ice(s) on Earth
- Important internal variables

✓ Rheological laws

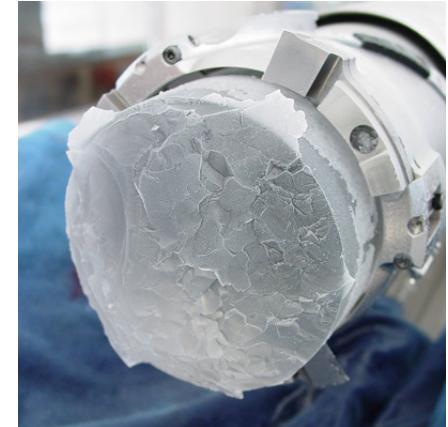
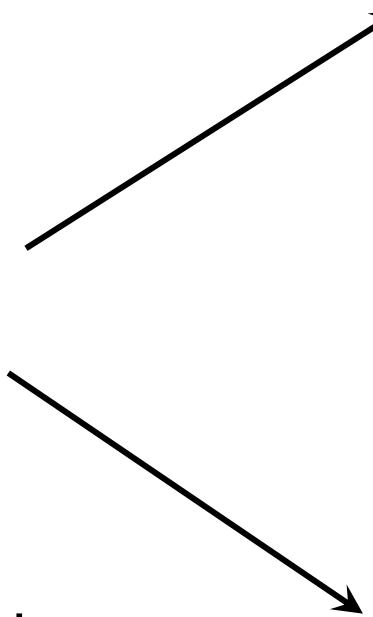
- Glen's flow law
- Anisotropic laws (GOLF and CAFFE)

Why studying ice flow?

Flow of ice



Help the interpretation of measurements
Dating of ice cores



Ice-sheets and Glaciers are
one part of the Climatic system

Surface Albedo

Mass balance Sea level



Ice: a fluid or a solid?



Fluid very very viscous at low strain rates



Solid elasto-brittle at high stresses

Both types of rheology at play simultaneously

Most ice flow models only account for the viscous component
(viscous model / visco-elastic model)

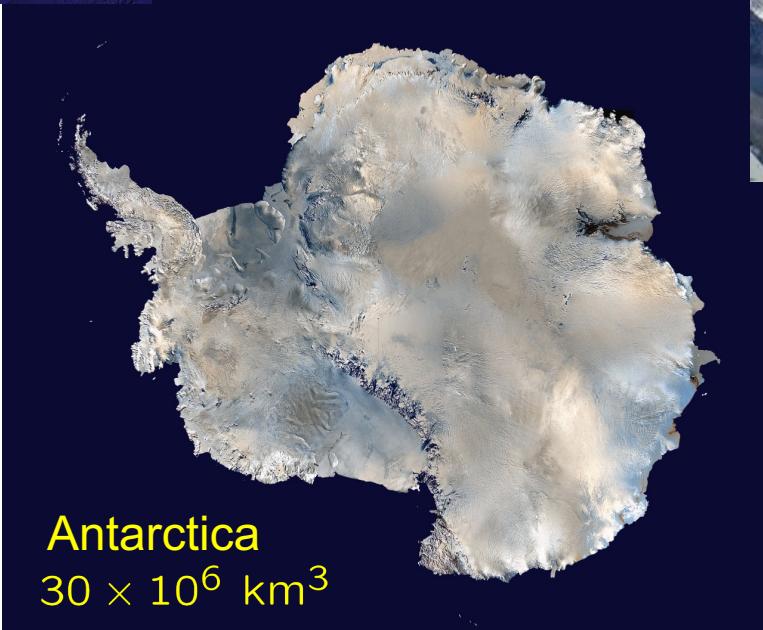
Flowing ice(s) on the Earth



Greenland
 $2 \times 10^6 \text{ km}^3$

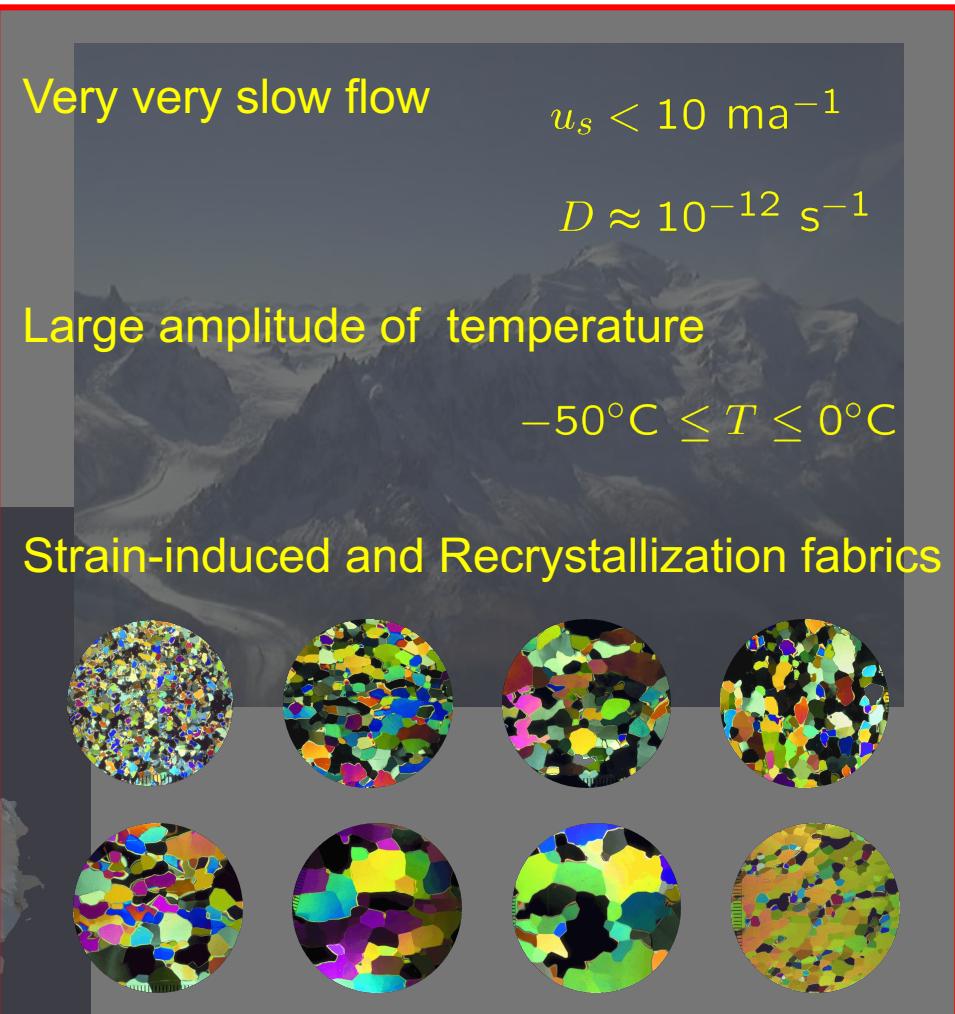
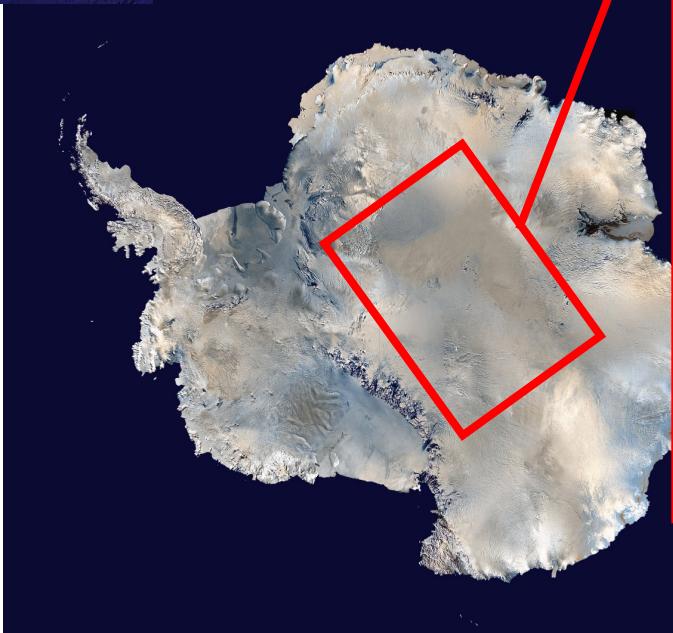
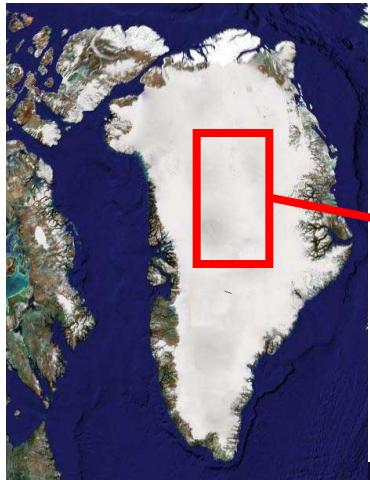


Glaciers (~220 000)
 $550 \times 10^3 \text{ km}^2$

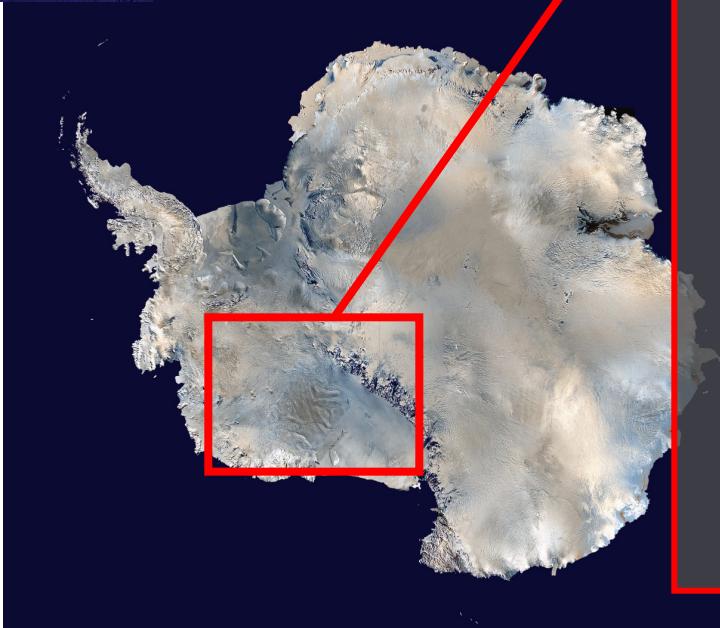
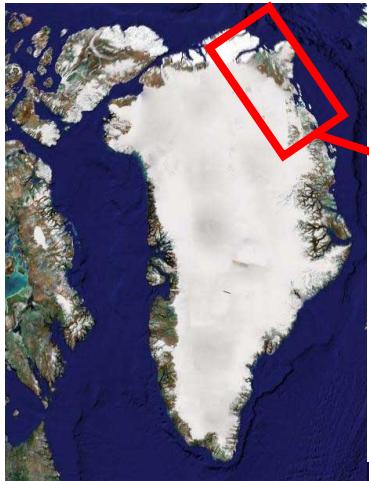


Antarctica
 $30 \times 10^6 \text{ km}^3$

Central part of ice-sheets



Margin of ice-sheets

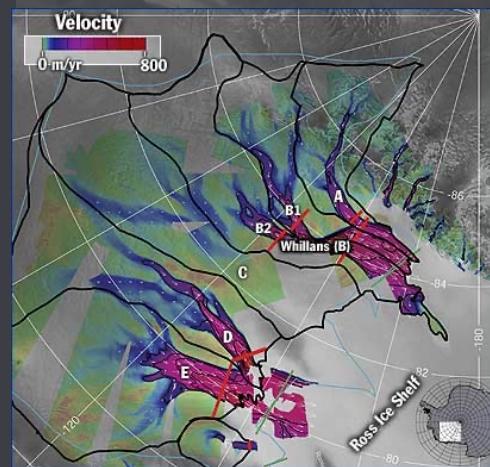


Faster flow $u_s \approx 1000 \text{ ma}^{-1}$

Higher temperature

Recrystallization fabrics

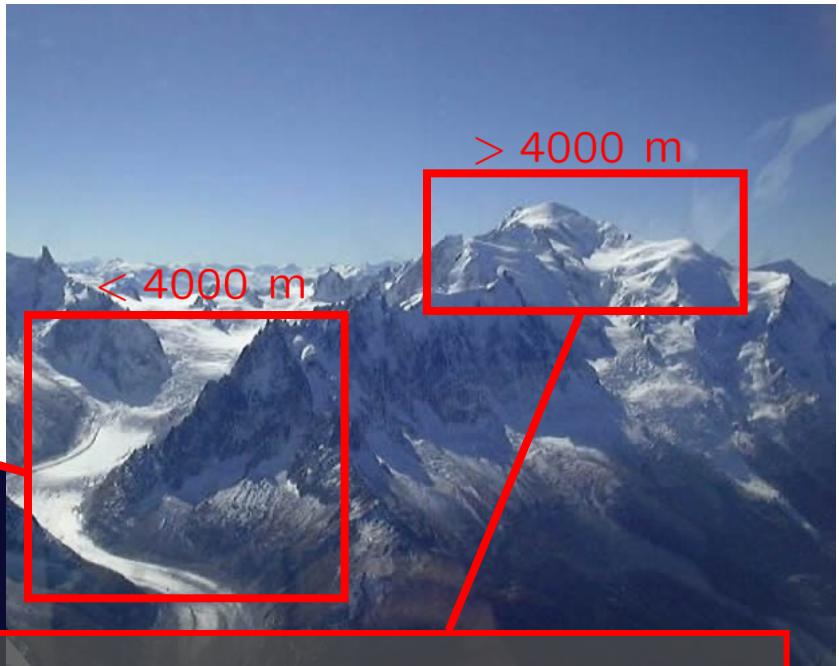
Major role of basal conditions



(Ian Joughin, JPL)

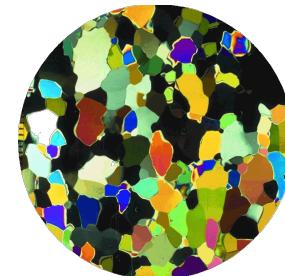
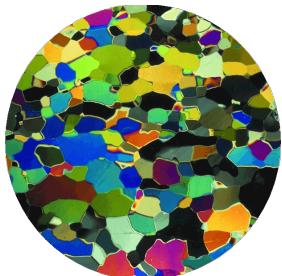
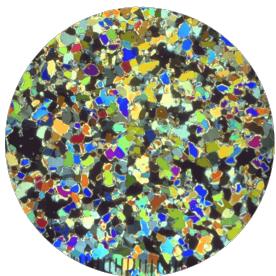
Glaciers

Faster flow $u_s \approx 100 \text{ ma}^{-1}$
Temperate ice
Stress-induced fabrics
Major role of basal conditions

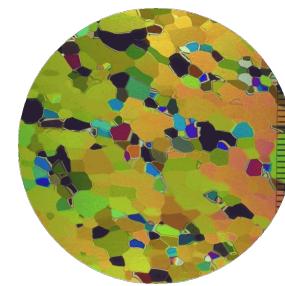
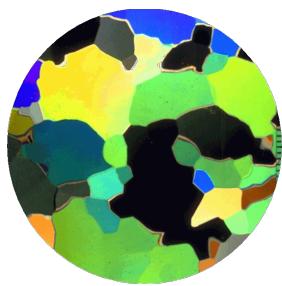
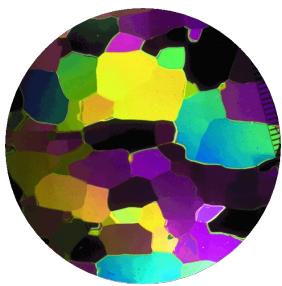
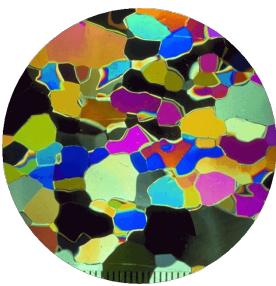


Slow flow $u_s \approx 10 \text{ ma}^{-1}$
Lower temperature $T < 0^\circ\text{C}$
Large part composed by snow/firn

Rheological properties of ice(s)



The behaviour of each piece of ice is unique !



Temperature

fabric

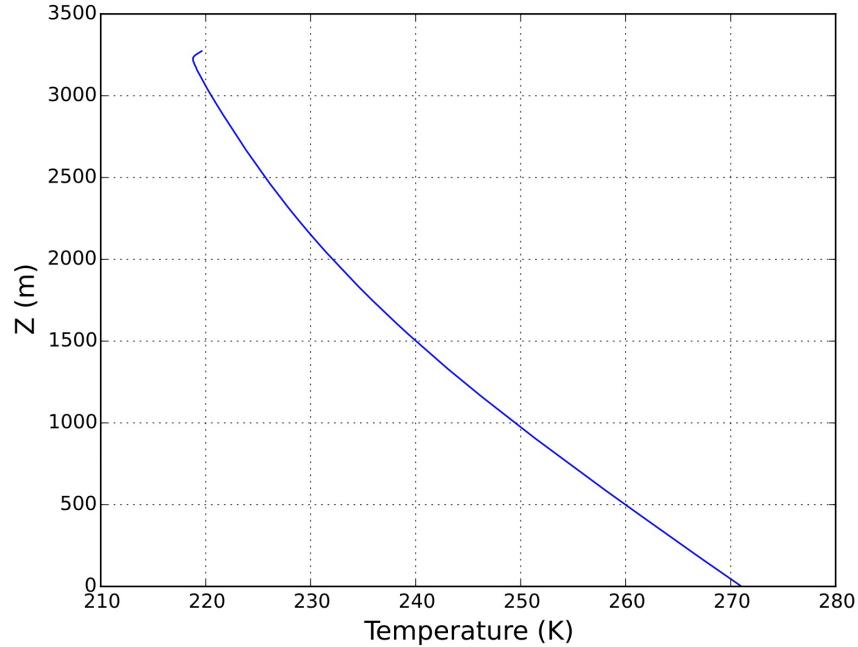
Size of the crystals (?)

Density

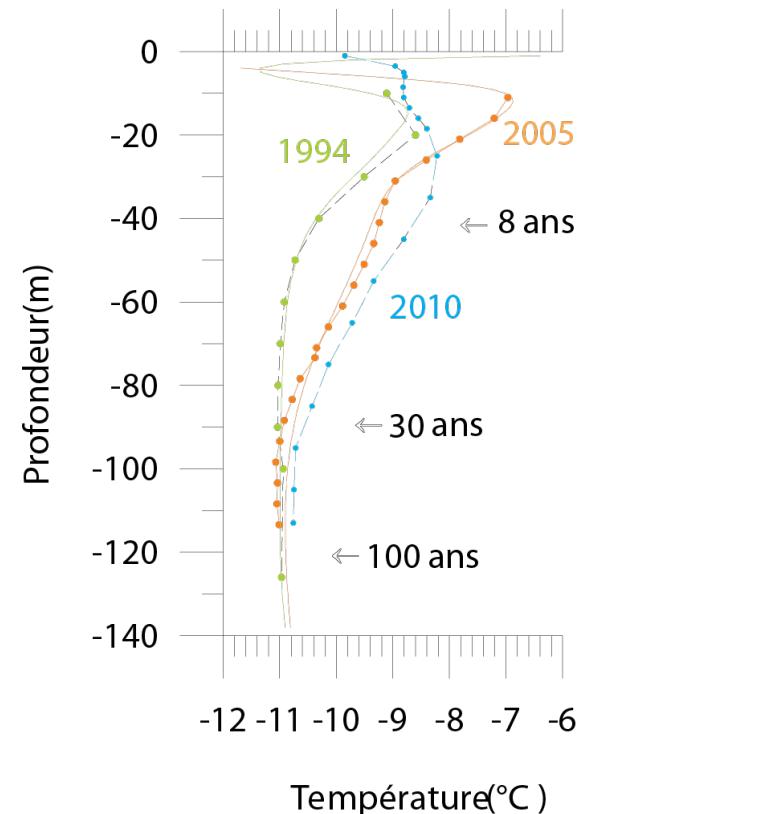
Water content

Dust content

Temperature evolution with depth



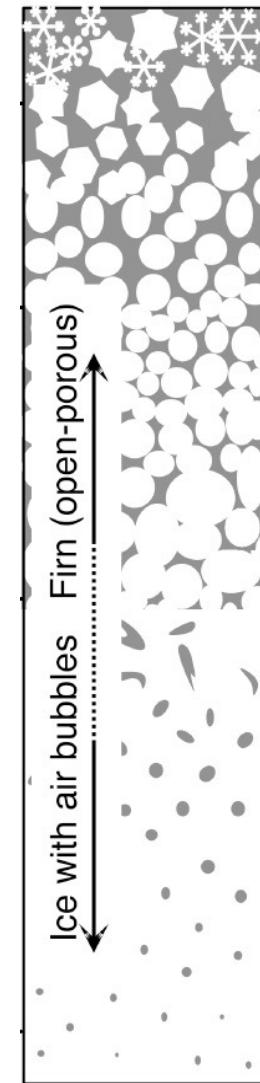
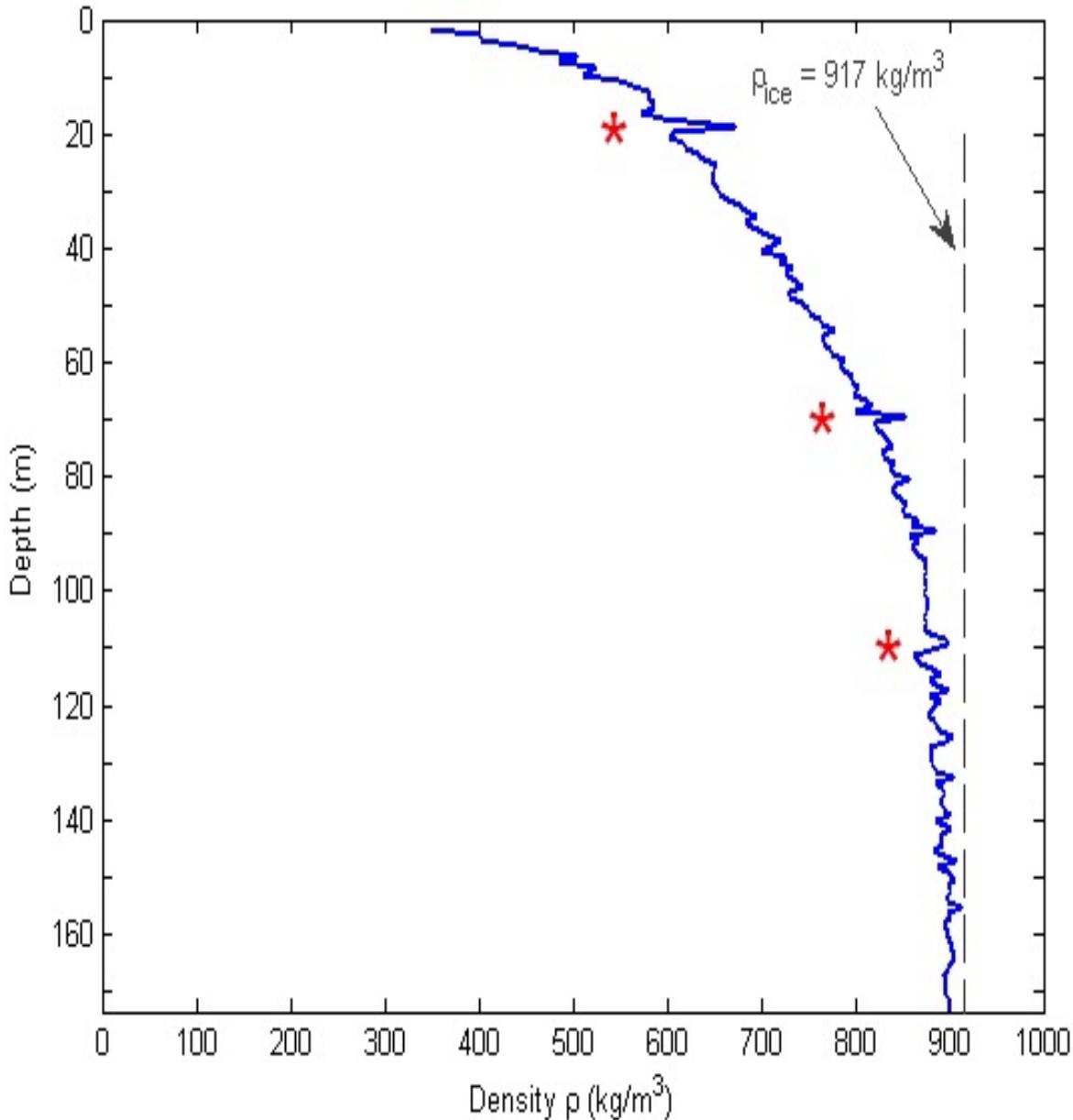
Temperature at Dome C (EPICA drilling), Antarctica



Temperature at Col du Dome (4300m), France

(*Gilbert et Vincent, GRL 2013*)

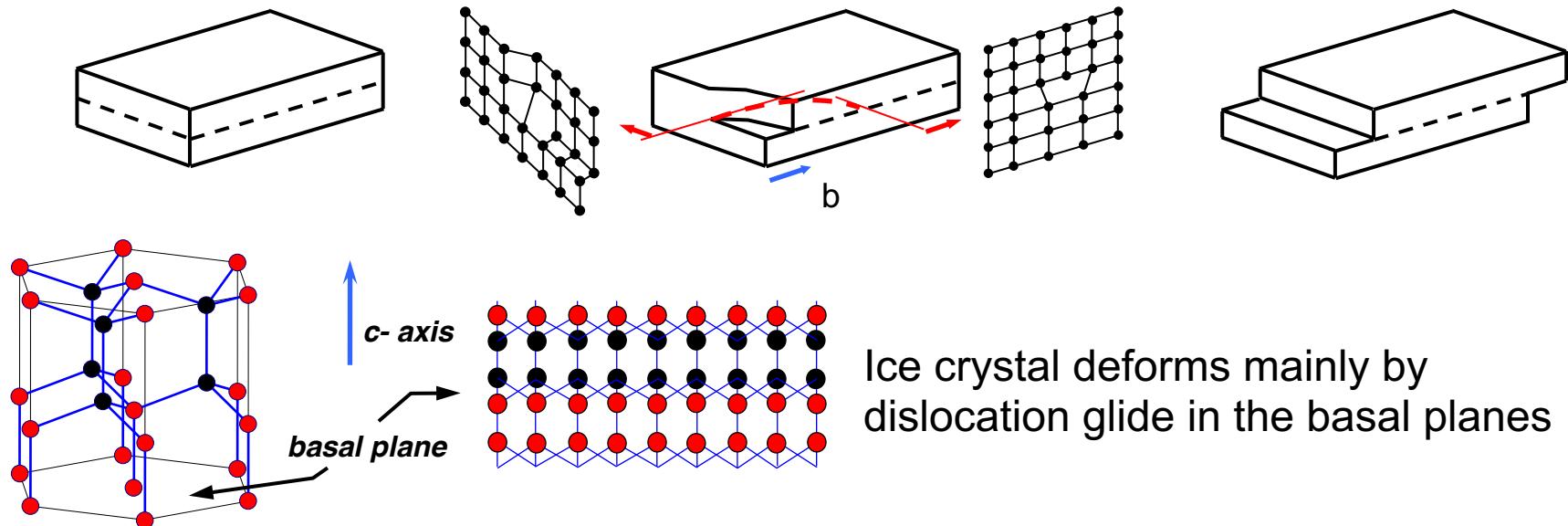
From snow to ice : density



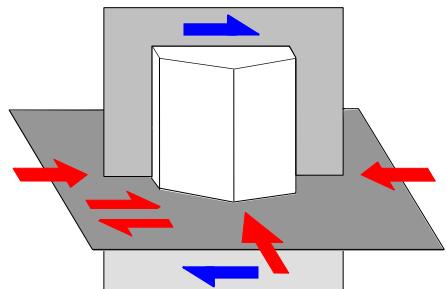
Schüpbach et al., 2016

Ice monocrystal viscoplastic behaviour

The viscoplastic deformation is due to the dislocation glide



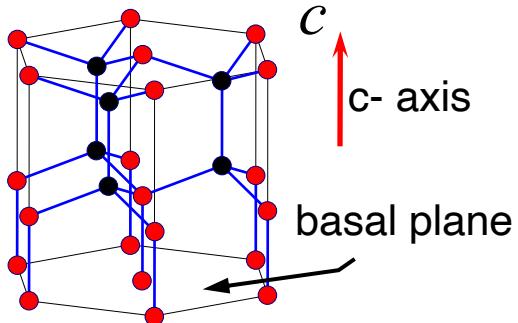
Ice is one of the most **anisotropic** natural material



Shearing parallel to basal plane is almost **1000 time faster** than compression (\perp ou \parallel p. b.) or shearing in the basal plane

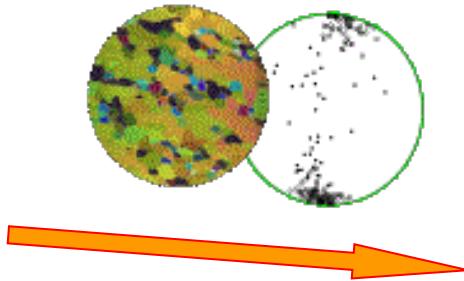
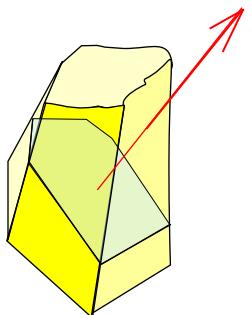
behave like a deck of cards

Polycrystalline anisotropy : ice fabric

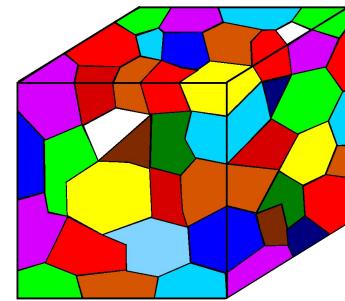


Hexagonal symmetry

Ice crystal



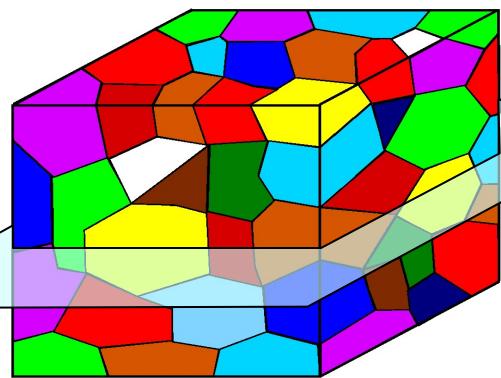
Fabric



Polycrystalline ice

Anisotropy function of the fabric

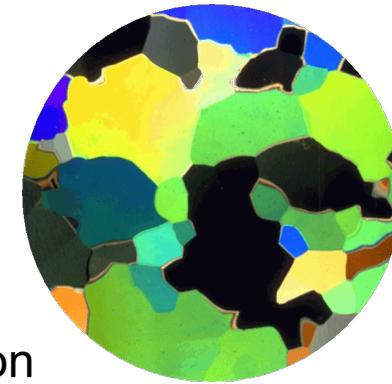
Fabric of polycrystalline ice



Thin section



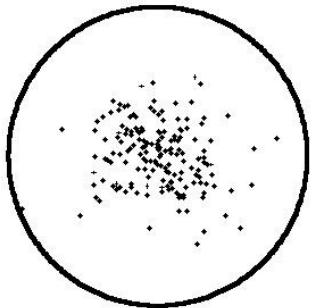
Under cross-polarized light



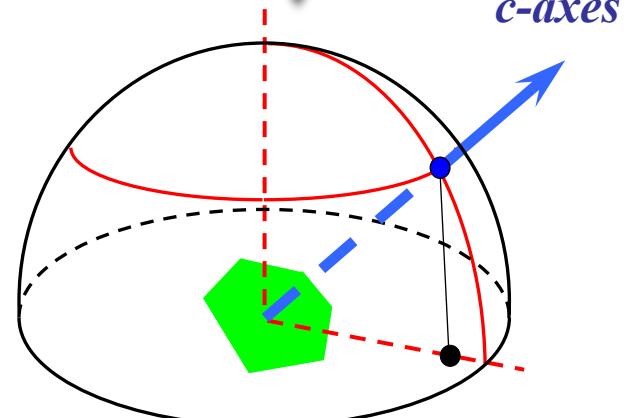
One color =
one grain =
one orientation



Schmidt diagram



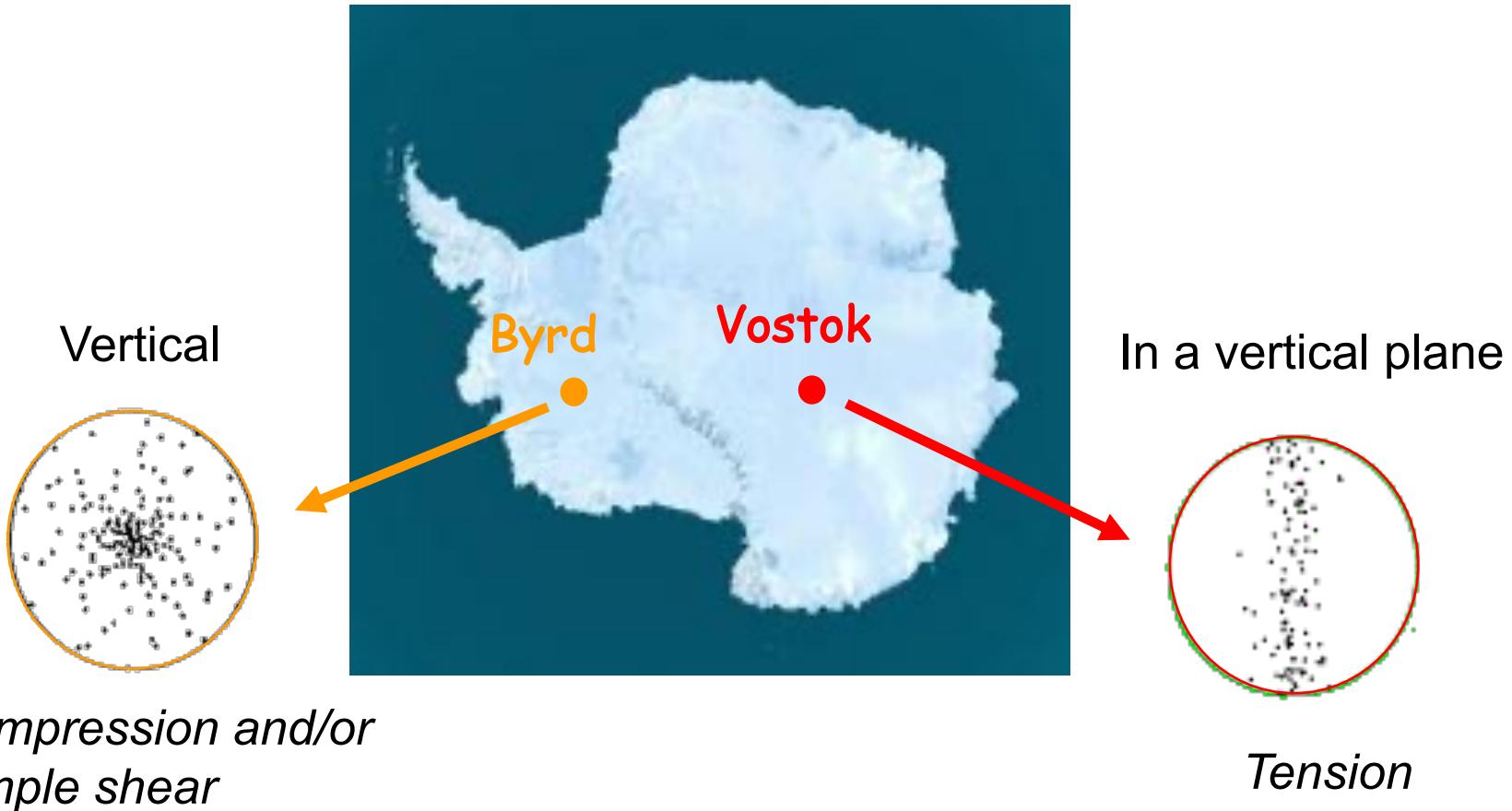
describe the fabric



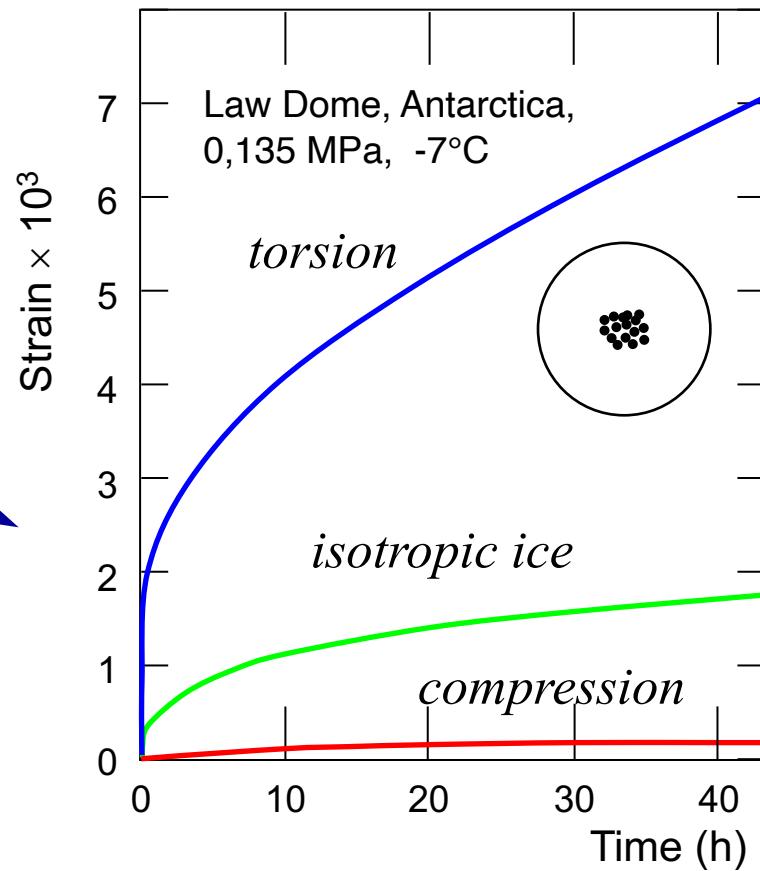
c-axes fabric

Observed ice fabric patterns

Depends on the strain history undergone by the polycrystal



Anisotropic polycrystalline ice



Single maximum fabric is about 10 time easier to shear than isotropic ice

Fractures in ice: damage



Damage allows to account in a continuous manner of discontinuous fractures in ice

Rheology of Ice(s)

✓ The Physics

- Ice(s) on Earth
- Important internal variables

✓ Rheological laws

- Glen's flow law
- Anisotropic laws (GOLF and CAFFE)
- A law for the firn/snow
- Associated evolution equations (fabric, density)

✓ Application

- Ice-Sheet / Ice-Shelf Enhancement Factor

Isotropic Ice (Glen's law)

Isotropic ice : Norton-Hoff type law

$$D_{ij} = EA\tau_e^{n-1}S_{ij} \quad \text{or} \quad S_{ij} = (EA)^{-1/n}I_{D_2}^{(1-n)/n}D_{ij}$$

where
$$\begin{cases} I_{D_2}^2 = D_{ij}D_{ij}/2 \\ \tau_e^2 = S_{ij}S_{ij}/2 \end{cases}$$

with

- Strain rate tensor: **D**
- Deviatoric stress tensor: **S**
- Glen's law fluidity parameter: A
- Enhancement factor: E
- Second invariant of strain rate tensor: I_{D_2}
- Second invariant of deviatoric stress tensor: τ_e

Viscosity dependency to temperature

$$D_{ij} = EA\tau_e^{n-1} S_{ij}$$

Arrhenius law for temperature dependency

$$A(T') = A(T_0) \exp \frac{Q}{R} \left(\frac{1}{T_0} - \frac{1}{T'} \right)$$

$T' = T - T_m$, with $T_m = 273.15 + 9.8 \times 10^{-8} p_i$ (Clausius-Clapeyron)

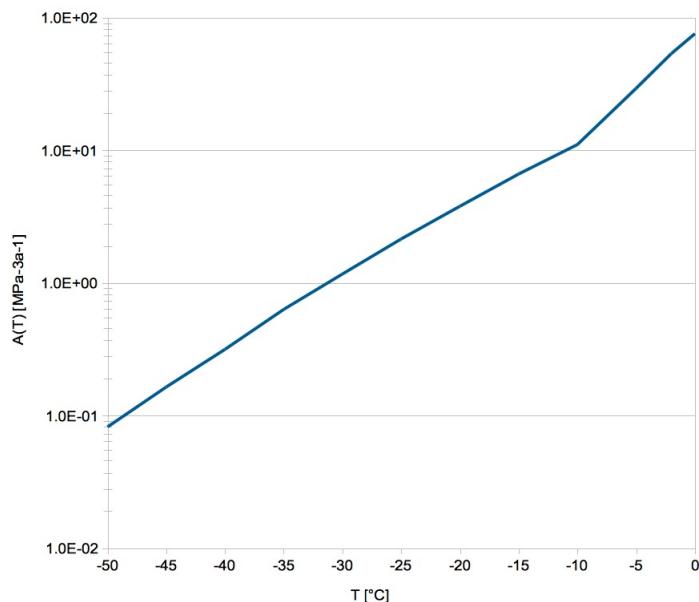
Q activation energy

$R = 8.314$ universal gaz constant

Viscosity dependency on temperature

Arrhenius law for temperature dependency

$$A(T') = A(T_0) \exp \frac{Q}{R} \left(\frac{1}{T_0} - \frac{1}{T'} \right)$$



$T [\text{°C}]$	$A [\text{Pa}^{-3} \text{s}^{-1}]$	$A [\text{MPa}^{-3} \text{s}^{-1}]$
0	$2.40\text{E}-024$	$7.574\text{E}+01$
-2	$1.70\text{E}-024$	$5.365\text{E}+01$
-5	$9.30\text{E}-025$	$2.935\text{E}+01$
-10	$3.50\text{E}-025$	$1.105\text{E}+01$
-15	$2.10\text{E}-025$	$6.627\text{E}+00$
-20	$1.20\text{E}-025$	$3.787\text{E}+00$
-25	$6.80\text{E}-026$	$2.146\text{E}+00$
-30	$3.70\text{E}-026$	$1.168\text{E}+00$
-35	$2.00\text{E}-026$	$6.312\text{E}-01$
-40	$1.00\text{E}-026$	$3.156\text{E}-01$
-45	$5.20\text{E}-027$	$1.641\text{E}-01$
-50	$2.60\text{E}-027$	$8.205\text{E}-02$

Recommended values by Cuffey and Patterson [2010]

3 orders of magnitude on A between -50°C and 0°C!

Evolution of temperature

If a cold glacier ($T < T_m$), one can solve only for temperature (conduction, advection)

$$\rho c_v \left(\frac{\partial T}{\partial t} + \mathbf{u} \cdot \operatorname{grad} T \right) = \operatorname{div}(\kappa \operatorname{grad} T) + \mathbf{D} : \boldsymbol{\sigma},$$

with $T < T_m$

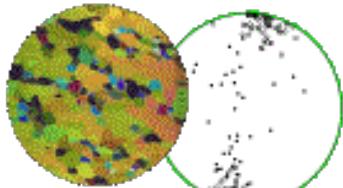
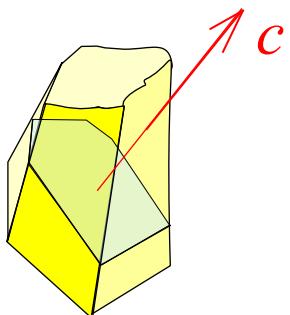
with

- heat conductivity: κ
- specific heat: c_v
- heat produced by deformation $\mathbf{D} : \boldsymbol{\sigma}$

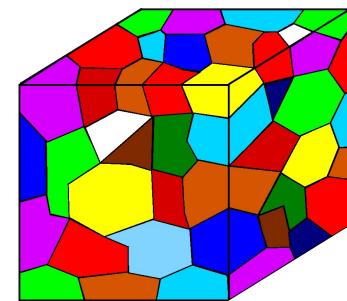
But for a polythermal glacier ($T \leq T_m$), better to solve for enthalpy
enthalpy = T for $T < T_m$, water content where $T = T_m$

Description of the fabric

Ice crystal



Fabric



Polycrystalline ice

Discrete: **two many variables**

Orientation tensors: **Yes!**

$$a^{(2)} = \langle \mathbf{c} \otimes \mathbf{c} \rangle$$

$$a^{(4)} = \langle \mathbf{c} \otimes \mathbf{c} \otimes \mathbf{c} \otimes \mathbf{c} \rangle$$



$$\begin{cases} a_1^{(2)} + a_2^{(2)} + a_3^{(2)} = 1 \\ a_1^{(2)}, a_2^{(2)}, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3 \end{cases}$$

Only 5 variables needed!

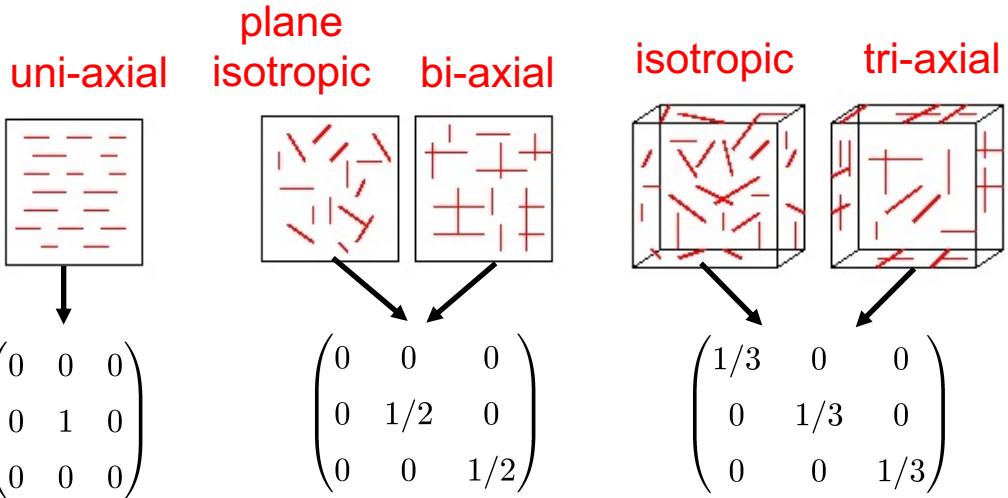
(The 2 eigenvalues and 3 eigenvectors of the second order orientation tensor)

Examples of fabric

Orientation tensors

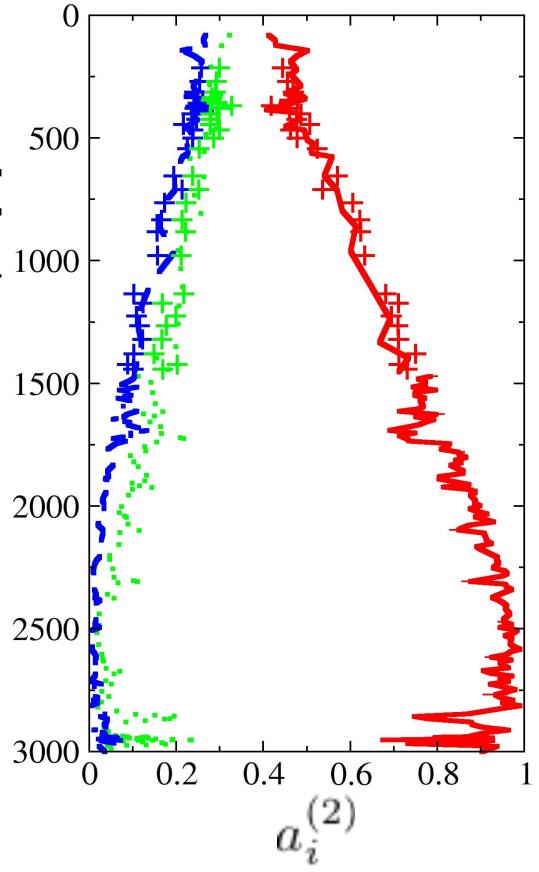
$$\mathbf{a}^{(2)} = \langle \mathbf{c} \otimes \mathbf{c} \rangle$$

$$\mathbf{a}^{(4)} = \langle \mathbf{c} \otimes \mathbf{c} \otimes \mathbf{c} \otimes \mathbf{c} \rangle$$

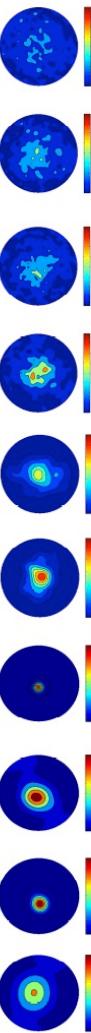


-  The knowledge of $\mathbf{a}^{(2)}$ is not sufficient
-  $\mathbf{a}^{(2)}$ and $\mathbf{a}^{(4)}$ are sufficient

Dome C (Antarctica)



(Wang et al, 2003 ; Durand et al., 2007)



Anisotropic Ice

Two dedicated laws in Elmer/Ice:

Name	GOLF	CAFFE
Anisotropy	Orthotropic	Enhancement factor
Collinear	No	Yes
Calibration	Tabulated using a micro-macro model	From experimental results

GOLF: General Orthotropic Flow Law [Gillet-Chaulet et al., 2005, 2006 ; Durand et al., 2009 ; Ma et al., 2010]

CAFFE: Continuum-mechanical, Anisotropic Flow model based on an anisotropic Flow Enhancement factor [Placidi and Hutter, 2006 ; Seddik et al., 2008, 2009 ; Placidi et al., 2010]

Anisotropic Ice

GOLF:

$$\sum_{r=1}^3 \left[\eta_r \text{tr}(\mathbf{M}_r \cdot \mathbf{D}) \mathbf{M}_r^D + \eta_{r+3} (\mathbf{D} \cdot \mathbf{M}_r + \mathbf{M}_r \cdot \mathbf{D})^D \right] = 2A\tau_e^{n-1}\boldsymbol{\tau}$$

$\eta_r = \eta_r(\mathbf{a}^{(2)})$, 6 relative viscosities function of the fabric

$\mathbf{M}_r = {}^o\vec{e}_r \otimes {}^o\vec{e}_r$, 3 structure tensors from the 3 principal axes

CAFFE:

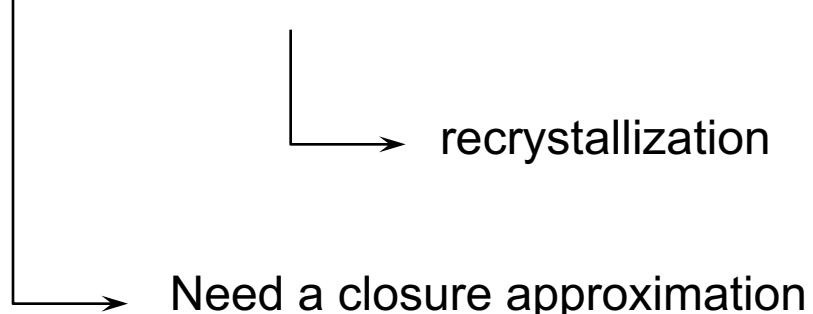
$$D = 2EA\tau_e^{n-1}\boldsymbol{\tau}$$

$E = E(\mathbf{a}^{(2)})$, 1 scalar enhancement factor function of the fabric

Fabric evolution

For both laws, need an equation describing the fabric evolution,
i.e. the evolution of $\mathbf{a}^{(2)}$

$$\frac{\partial \mathbf{a}^{(2)}}{\partial t} + g(\mathbf{S}, \mathbf{D}, \mathbf{a}^{(2)}, \mathbf{a}^{(4)}) + \kappa(\mathbf{I} - \mathbf{a}^{(2)}) = 0$$



$$\mathbf{a}^{(4)} = f(\mathbf{a}^{(2)})$$

Only Macroscopic quantities

[Gödert, 2003 ; Gillet-Chaulet et al., 2006]

Equations to be solved (Anisotropy)

➤ Velocities

Stokes Equations

$$\begin{cases} \operatorname{div} \boldsymbol{\sigma} + \rho \mathbf{g} = 0 \\ \operatorname{div} \rho \mathbf{u} = 0 \end{cases}$$

+ behaviour (GOLF, CAFFE)

- Inputs

- Fabric
- Temperature
- Geometry

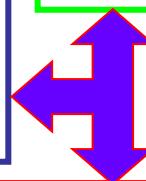
- Outputs

- Velocities and isotropic pressure
- Stresses, strain-rates

➤ Fabric

$$\frac{\partial \mathbf{a}^{(2)}}{\partial t} + g(\mathbf{S}, \mathbf{D}, \mathbf{a}^{(2)}, \mathbf{a}^{(4)}) + \kappa(\mathbf{I} - \mathbf{a}^{(2)}) = 0$$

- Inputs : Velocities, stresses, strain-rates, rotation rate
- BC : Isotropic ice at the surface
- Outputs : Fabric field



Fully coupled ϵ_C

➤ Free surface elevation

$$\frac{\partial h}{\partial t} + \frac{\partial h}{\partial x_1} u_1 + \frac{\partial h}{\partial x_2} u_2 - u_3 = b$$

- Inputs Velocities
- Outputs Surface elevation

What still to be done?

We are currently less active at IGE on that field (anisotropy)

But regain of interest on that topic regarding:

- the influence of ice rheology on shear margin in Antarctica

e.g.: *Lilien et al., 2021. Modeling ice-crystal fabric as a proxy for ice-stream stability. Journal of Geophysical Research: Earth Surface, 126, e2021JF006306, doi: 10.1029/2021JF006306*

- the location of very old ice in Antarctica

e.g.: *Passalacqua et al., 2018. Brief communication: Candidate sites of 1.5 Myr old ice 37 km southwest of the Dome C summit, East Antarctica, The Cryosphere, 12, 2167-2174, doi:10.5194/tc-12-2167-2018*

A unified rheological model including all internal variables: temperature/water content, density, anisotropy, damage...

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Ice, glacier and ice-sheet flow

- Introduction: Cryosphere and climate change
- Ice(s), a material with a complex rheology
- **Glaciers and Risks in a warming climate**
- Grounding line and friction

Glaciers & warming climate

When temperature increases...

- ... summer melt is increasing (more water available)
- ... altitude of equilibrium line is increasing
- ... glacier fronts are retreating (good location for lakes!)
- ... glaciers are warming? ... not so easy...**

Glaciers & warming climate

When a glacier warms...

... its viscosity decreases, it flows faster

... it potentially switches from cold impermeable ice to temperate permeable ice

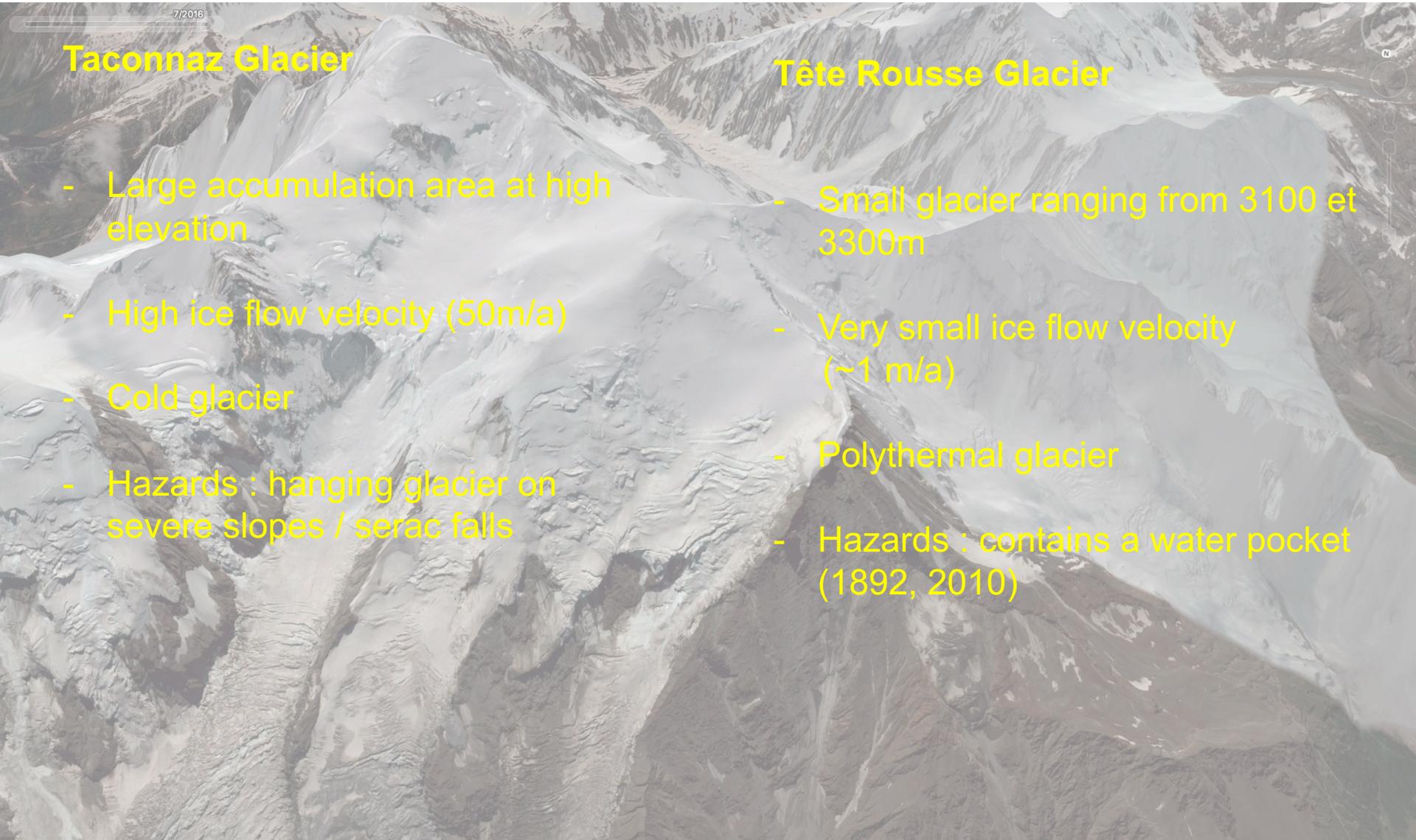
... potential switch from a cold base glued to the bedrock to a temperate base allowing sliding

... **risk is increasing?** ... not so easy...

Two examples, two risk evolutions



So close, but so different



Taconnaz Glacier

- Large accumulation area at high elevation
- High ice flow velocity (50m/a)
- Cold glacier
- Hazards : hanging glacier on severe slopes / serac falls

Tête Rousse Glacier

- Small glacier ranging from 3100 et 3300m
- Very small ice flow velocity (~1 m/a)
- Polythermal glacier
- Hazards : contains a water pocket (1892, 2010)

Tools for risk management

Instrumentation

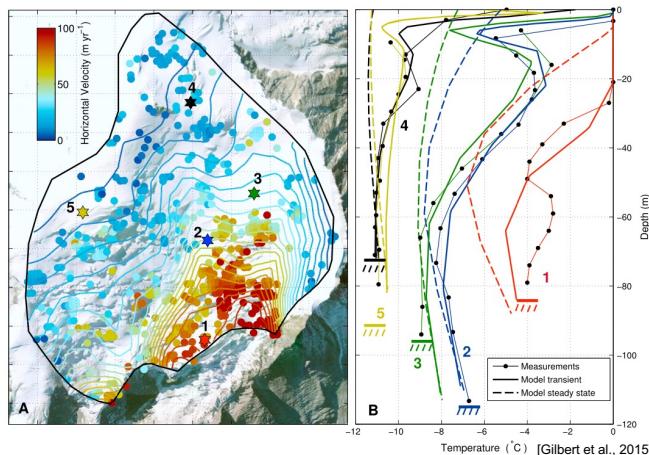


@Bruno Jourdain

- Ice temperature glace (*thermistors in drills*)
- Ice deformation (*Inclinometers in drills*)
- Surface DEM (*drone, lidar*)
- Bedrock DEM (*radar*)
- Surface velocity (*stakes, satellites*)

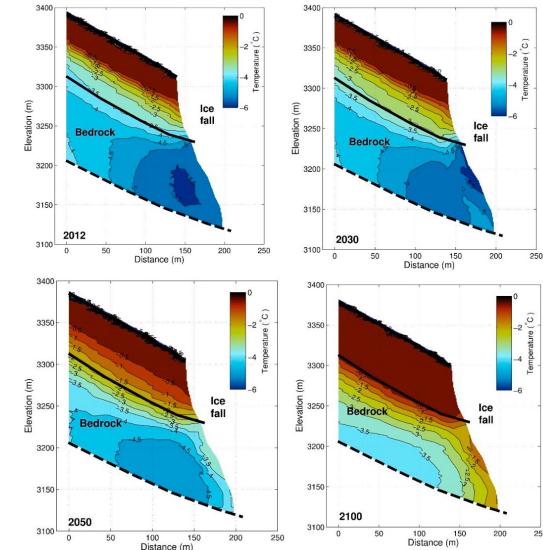


- Numerical modelling coupling *thermodynamics, damage, firm rheology, percolation/refreezing of surface melt...*
- Calibration et validation of model



Model / data - past / present

Future evolution

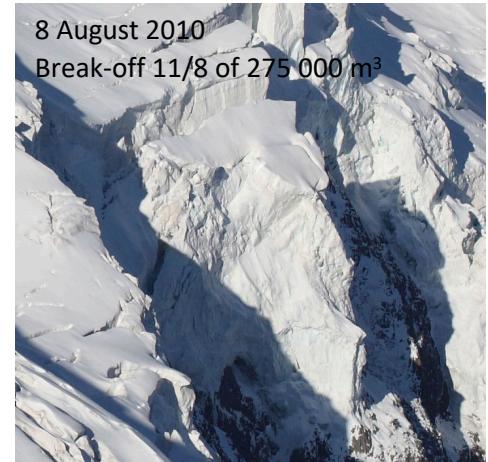


- Model evolution for different climatic scenarios
- Identification of key processes

Taconnaz context

Regular break-off of seracs triggering large avalanches (21% of avalanches)

From 1900 to 2000 : 75 observed avalanches



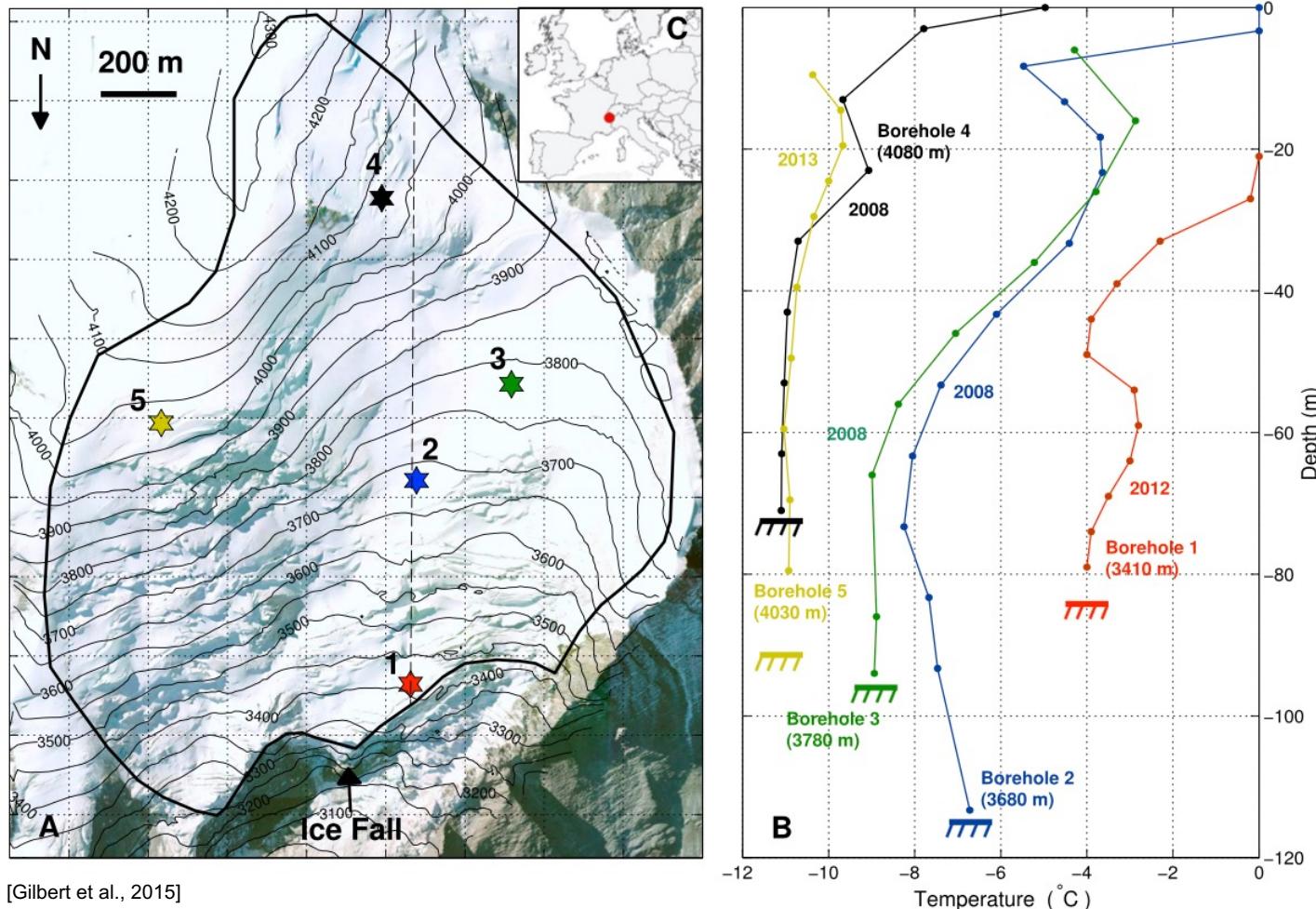
2006 : partial destruction of the protection structure



Avalanche 15 April 2015



Current state from observation

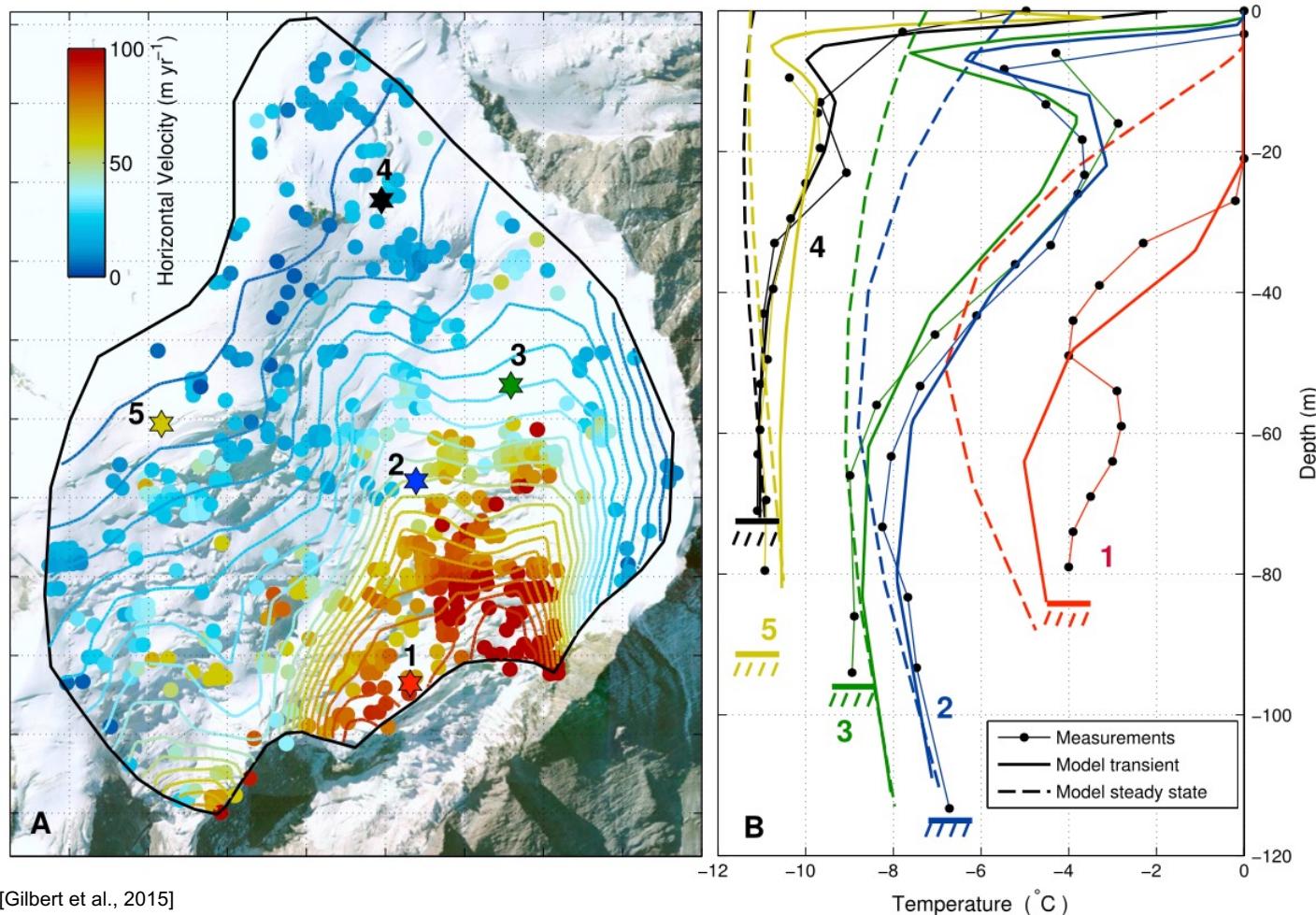


[Gilbert et al., 2015]

Some areas not far from ice melting point at the base

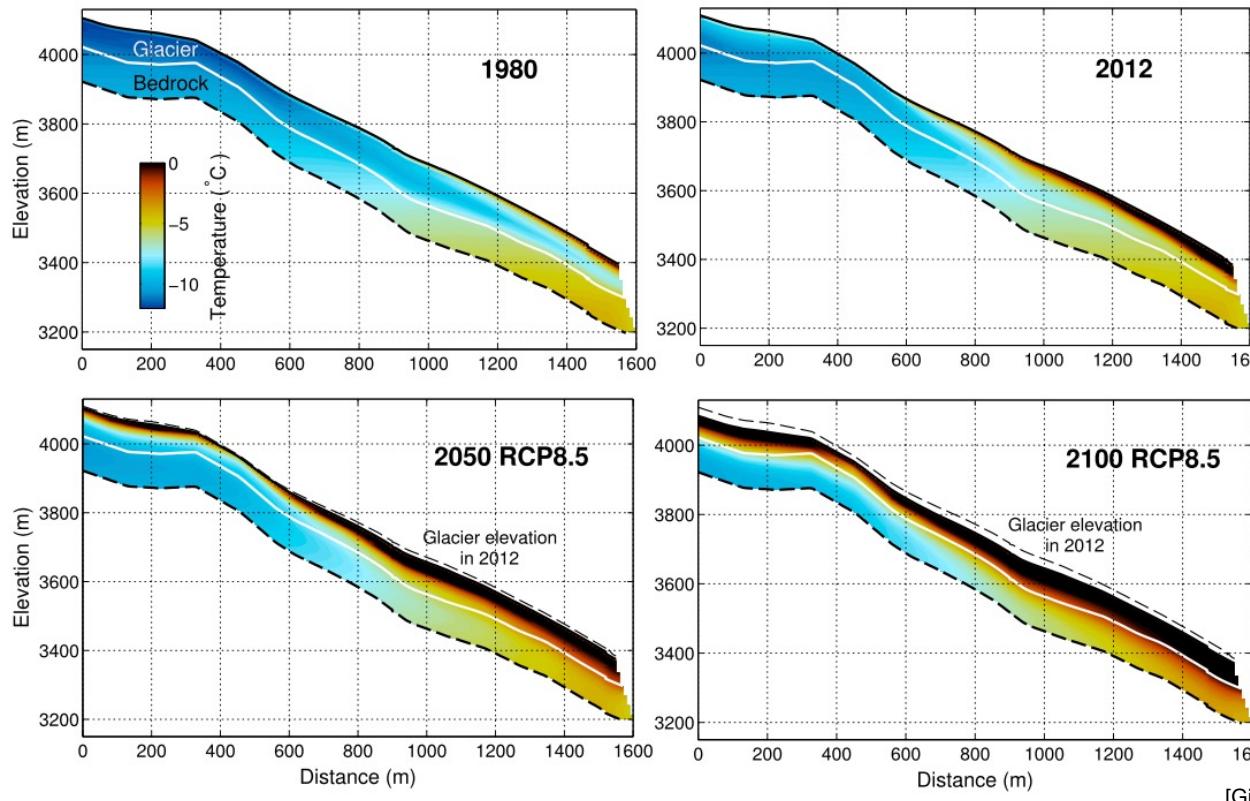
How will evolve the temperature field in the future?

Model validation against observations (current state)



[Gilbert et al., 2015]

Evolution in the future (RCP8.5)



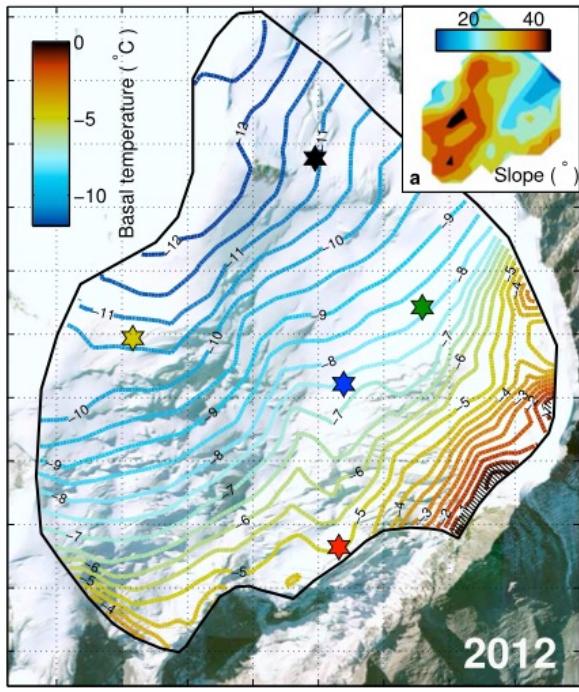
[Gilbert et al., 2015]

When climate is warming...

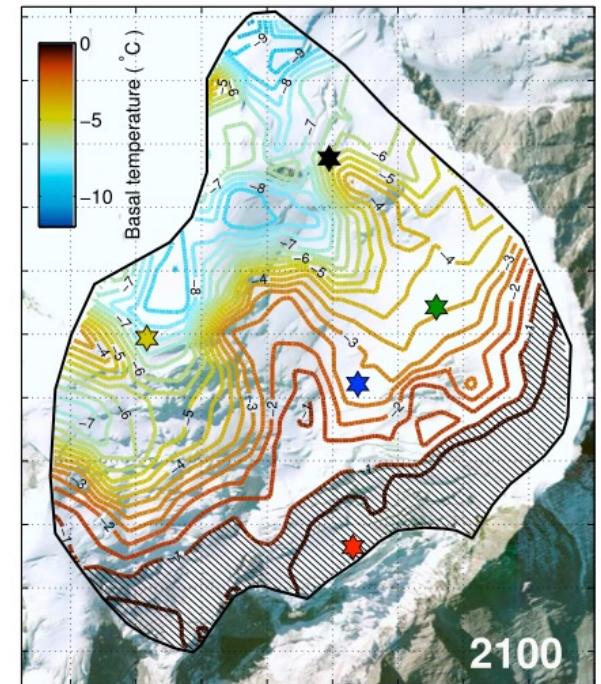
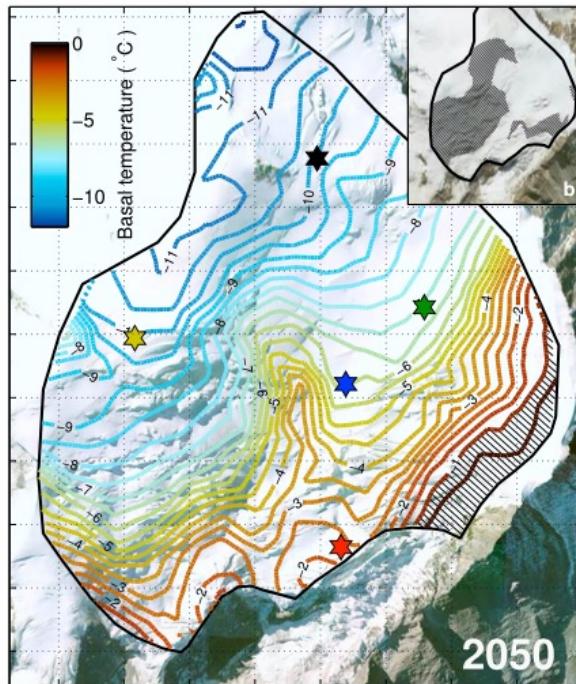
... the glacier is warming
... the hazards from its front destabilisation is increasing

Evolution in the future (RCP8.5)

Taconnaz



[Gilbert et al., 2015]



Propagation from the front of a temperate area at the base of the glacier

Progressive retreat of the front in the future?

OR

Destabilisation of a large part of the glacier at once?

→ important to continue a temperature survey measurements in the future

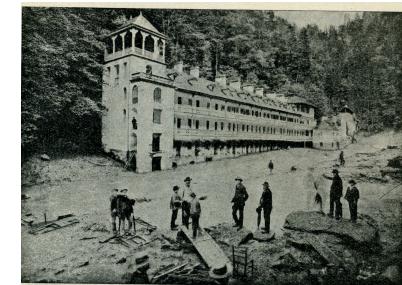
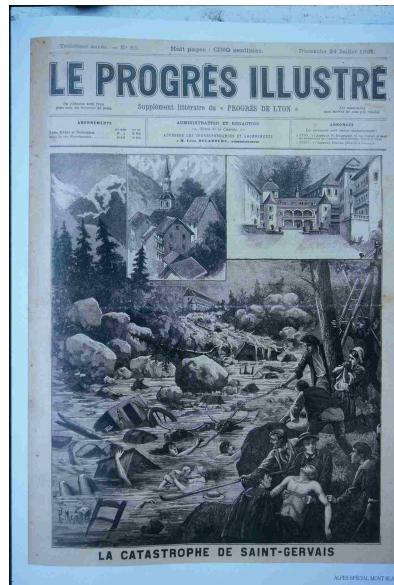
Tête Rousse context

11 July 1892

175 fatalities

200 000 m³ water + ice

→ Flood produced 800 000 m³ of sediment

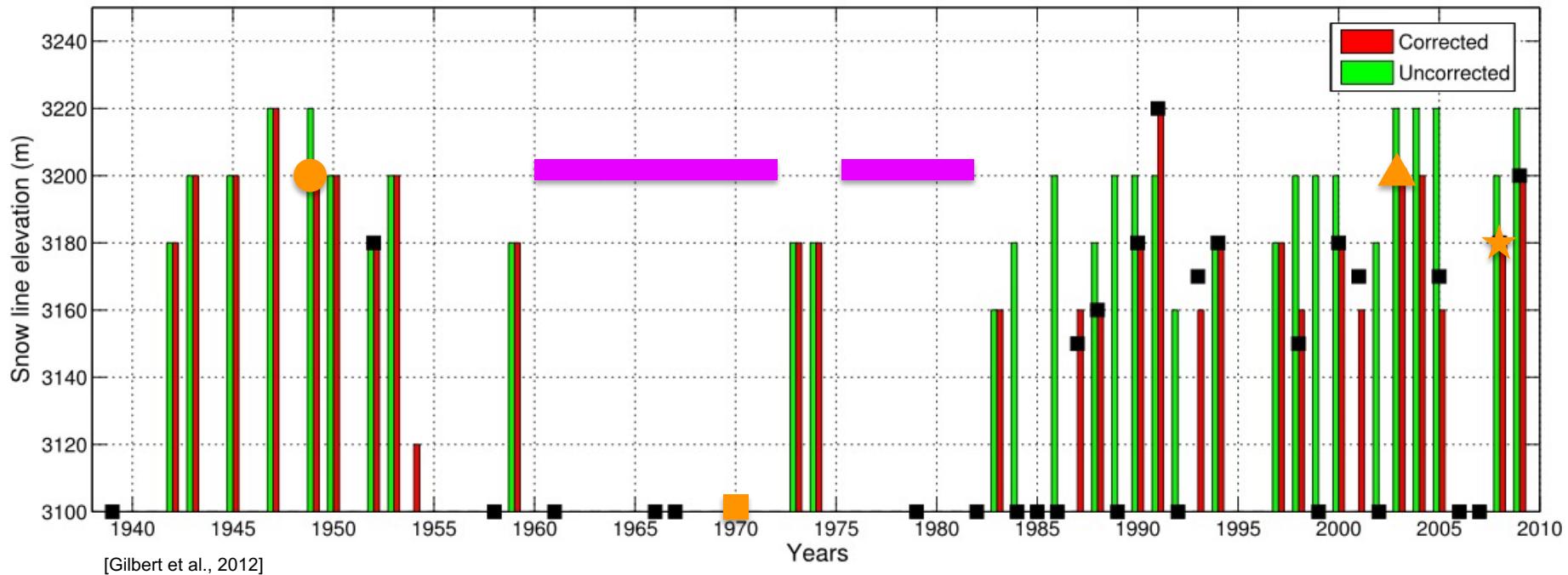


2010-today

A new water pocket found in 2010
~60 000 m³ of water

Artificially drained in 2010, 2011 and 2012

Influence of snow cover on temperature

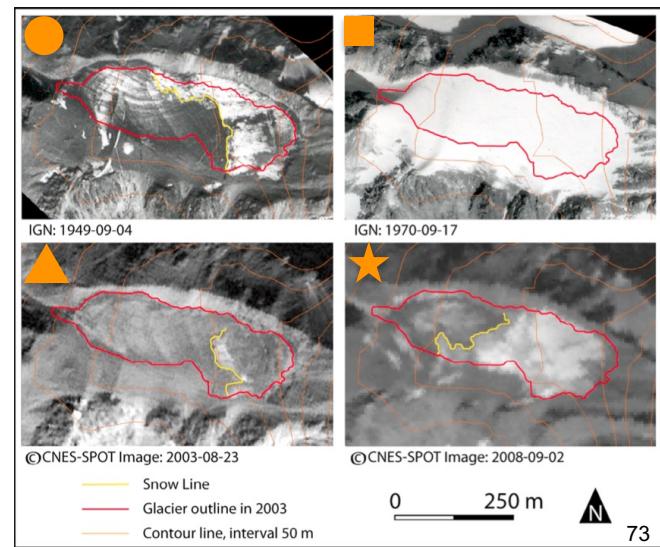


When the glacier is **covered** by snow and firn...

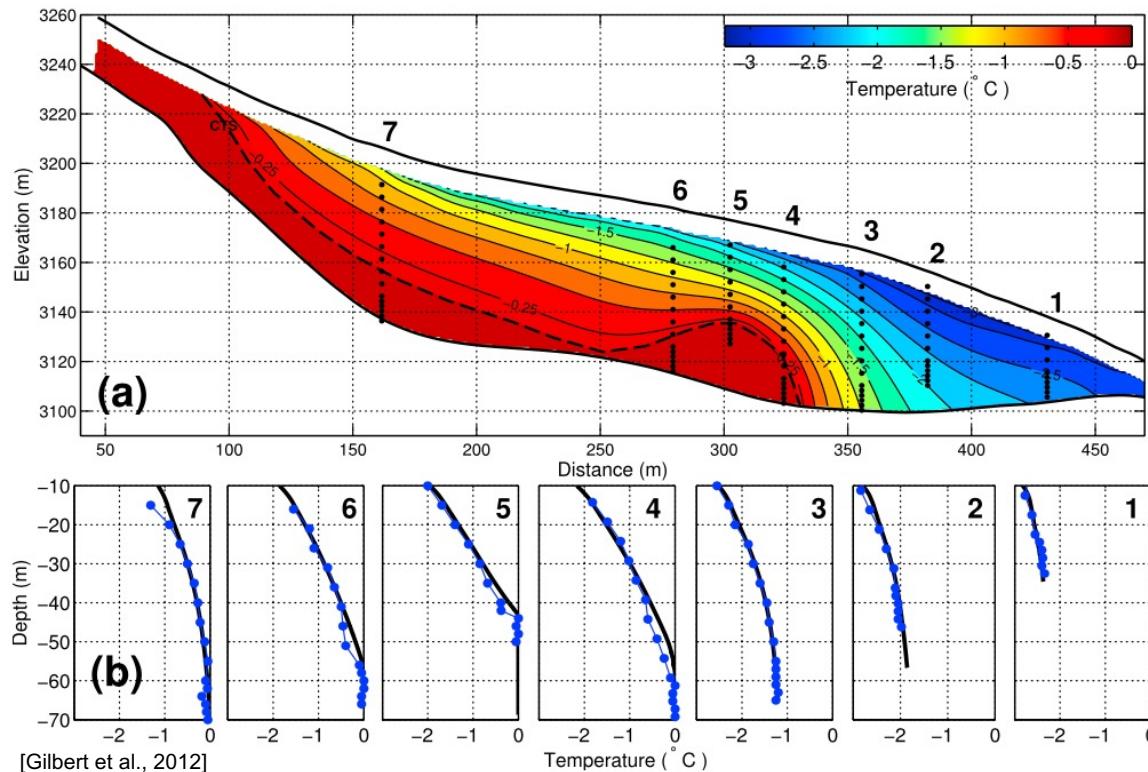
... melt water can percolate and refreeze,
releasing heat at depth

... the glacier is insulated from the cold
temperature of winter

Strong variability of snow cover since 1940



Observation / model validation (2010 state)



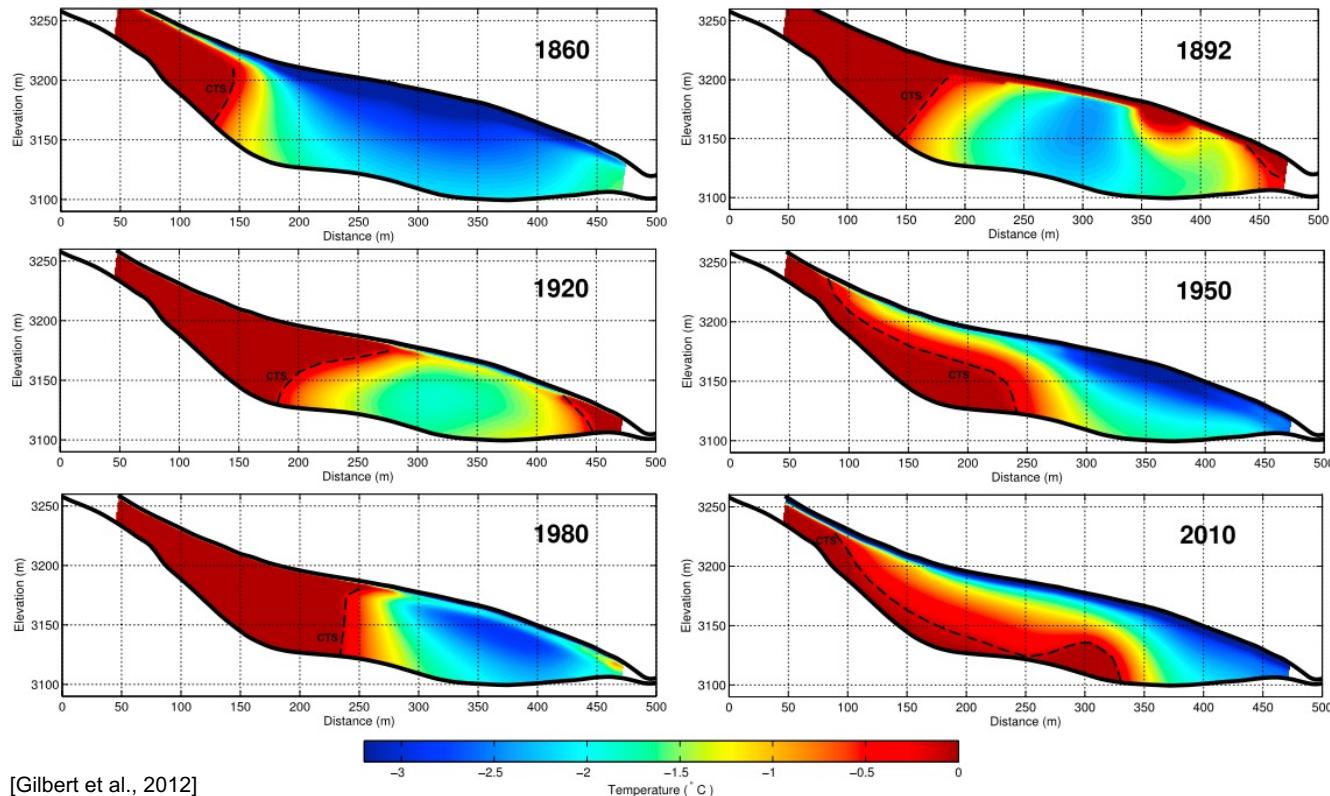
Glacier front is cold, upstream parts are temperate

At the top, larger snow cover: refreezing + insulation induce a warming of the ice

At the front, less snow cover: runoff of melt + low insulation during winter induce a cool down!

The temperature field indicates that the water pocket has formed in the 80's!

Temperature reconstruction since 1860



[Gilbert et al., 2012]

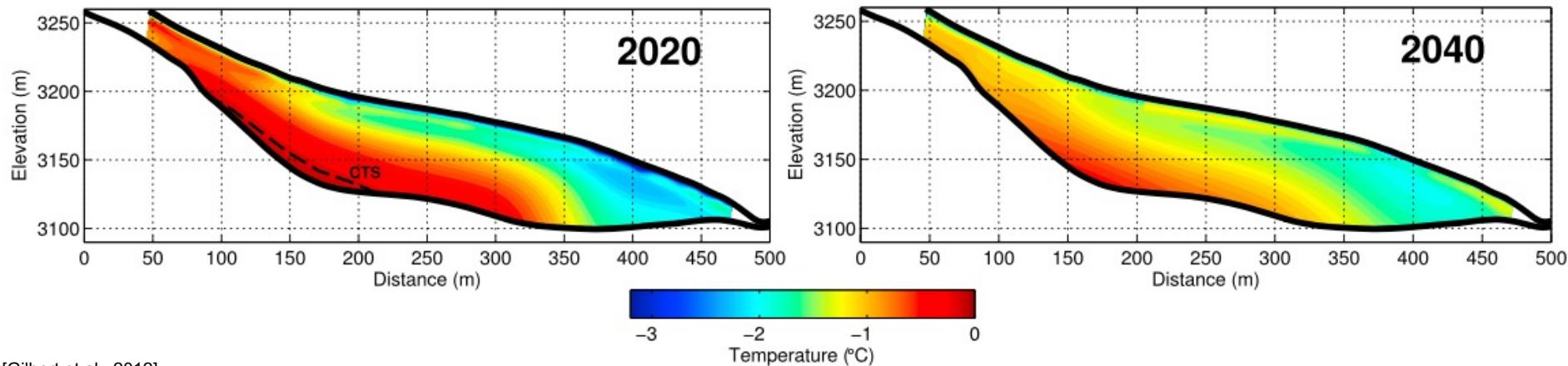
Ice flow and ice thickness very low

fast evolution of the temperate area (~20 years)

climatic warming induces a decrease of the temperate area

In the future?

Scenario: + 2°C and constant accumulation



[Gilbert et al., 2012]

When climate is warming...

... the glacier is cooling down

... the hazards from presence or breakoff of the water pocket is decreasing

Two examples that show a very contrasted evolution of risk in the context of a warming climate

Necessity to model the thermo-mechanics behaviour of glaciers when assessing glacial hazards

Necessity to have an approach coupling data and model to access the hazard evolution

- Ice, glacier and ice-sheet flow
- Introduction: Cryosphere and climate change
- Ice(s), a material with a complex rheology
- Glaciers and Risks in a warming climate
- **Grounding line and friction**

Which equations? Which boundary conditions?

Stokes:

$$\begin{aligned}\operatorname{div} \sigma + \rho g &= 0 \\ \operatorname{div} u &= 0\end{aligned}$$

Flow law:

$$S = f(D)$$

Basal sliding: $\tau_b = f(u)$

Upper free surface:

$$\frac{\partial h}{\partial t} + \mathbf{u} \cdot \frac{\partial z_s}{\partial \mathbf{x}} - w = a$$

$$f(\mathbf{x}_f) = 0$$

Ice-shelf

Lower free surface:

$$\frac{\partial h}{\partial t} + \mathbf{u} \cdot \frac{\partial z_b}{\partial \mathbf{x}} - w = b$$

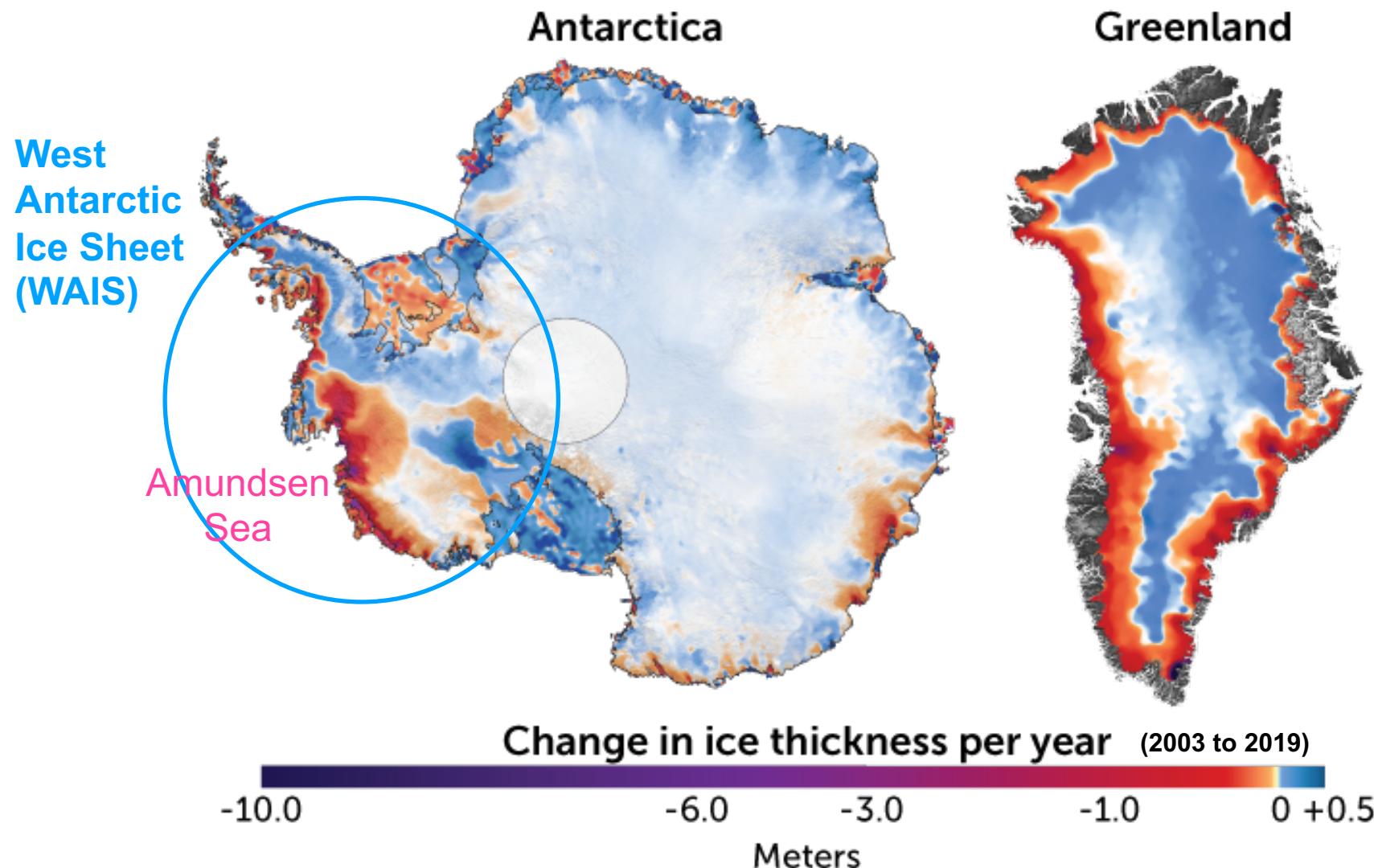
Grounded ice

Floating ice

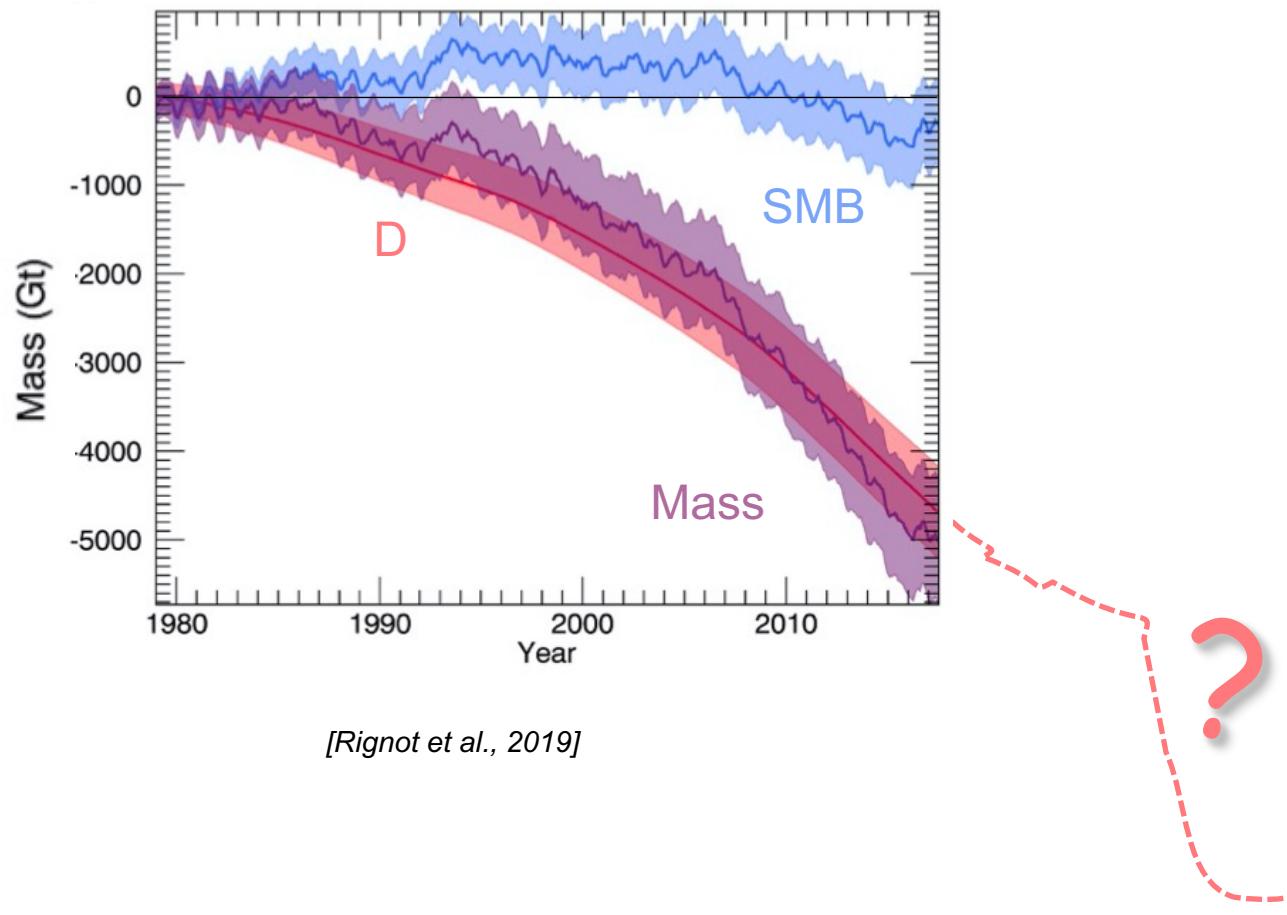
$$f(\mathbf{x}_g) = 0$$

Grounding line

Increasing mass loss contributing to sea level rise



Future Ice Discharge



→ What will be the future contribution of Ice Discharge for the next centuries?
Need accurate description of the Grounding Line dynamics

Potential processes at play



Changes at the ice/ocean front

- increased calving rate
- decreased of melange stiffness

Changes at the base (grounded)

- decreased basal friction (zwally effect)

Changes at the lateral margin

- decreased lateral friction (water in crevasses)

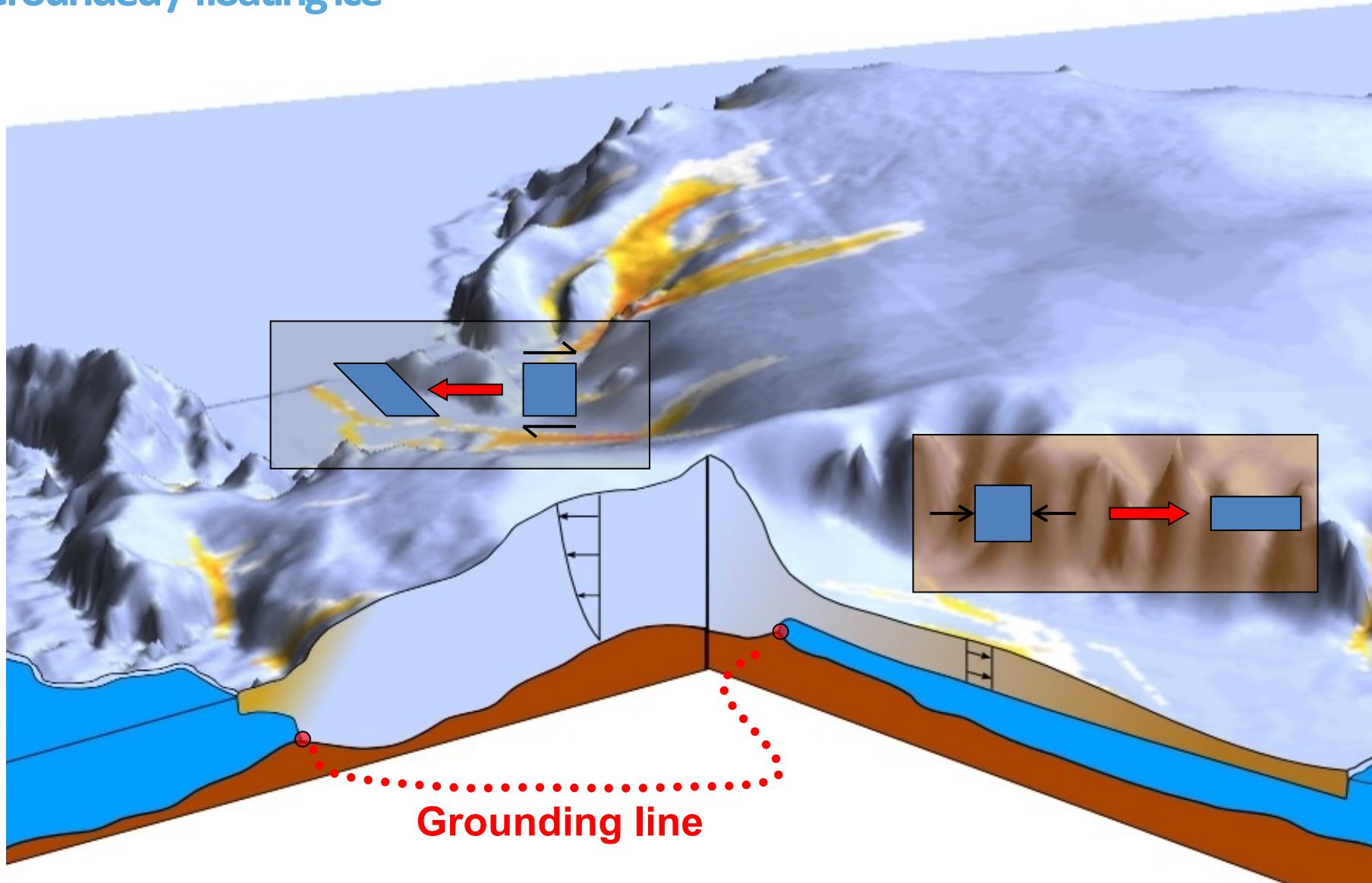
Changes in the ice rheology

- refreezing of runoff in the firn...

Changes at the base (floating)

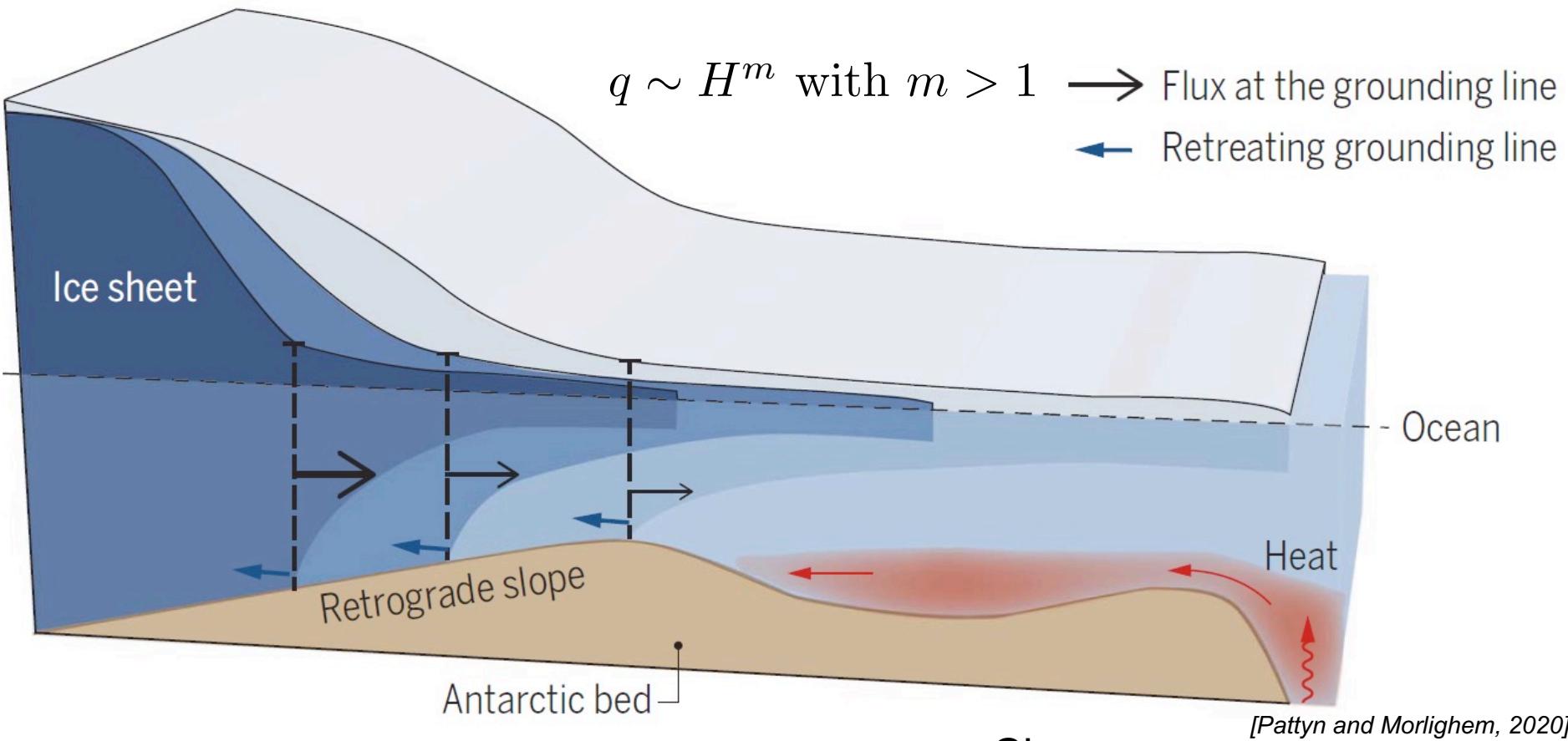
- reduced buttressing (increase of basal melt)

Grounded / floating ice

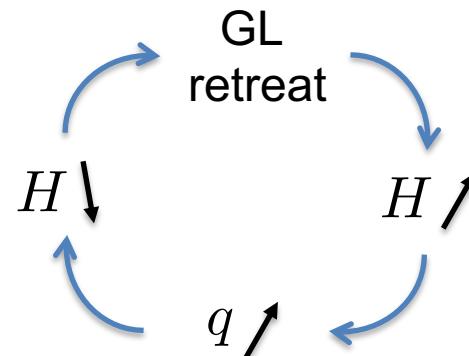


Contrasted state of stress (illustrated after for 2d flow)

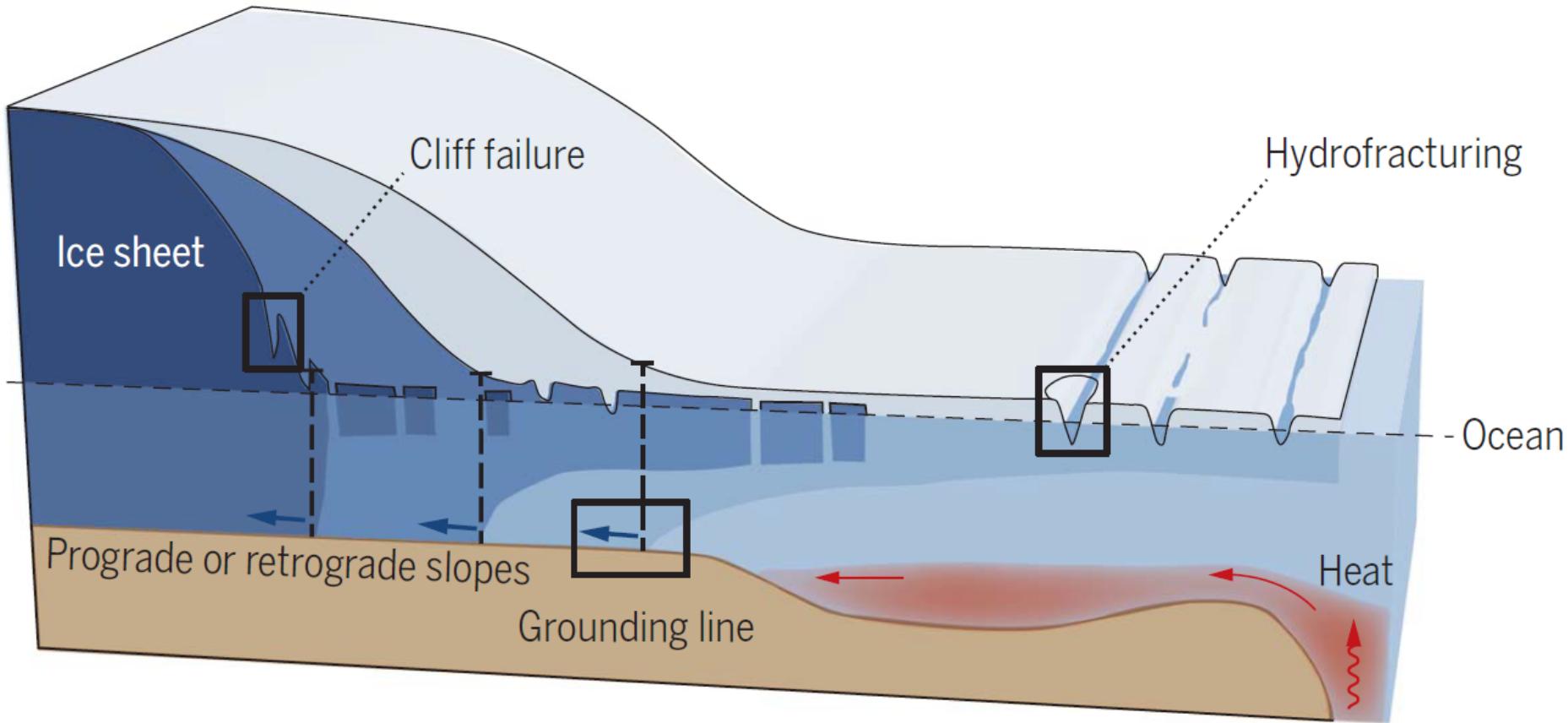
Marine Ice Sheet Instability (MISI)



Positive feedback
No stable position on retrograde slopes



Marine Ice Cliff Instability (MICI)



No clear consensus – only one ice-sheet model including this process for AR6!

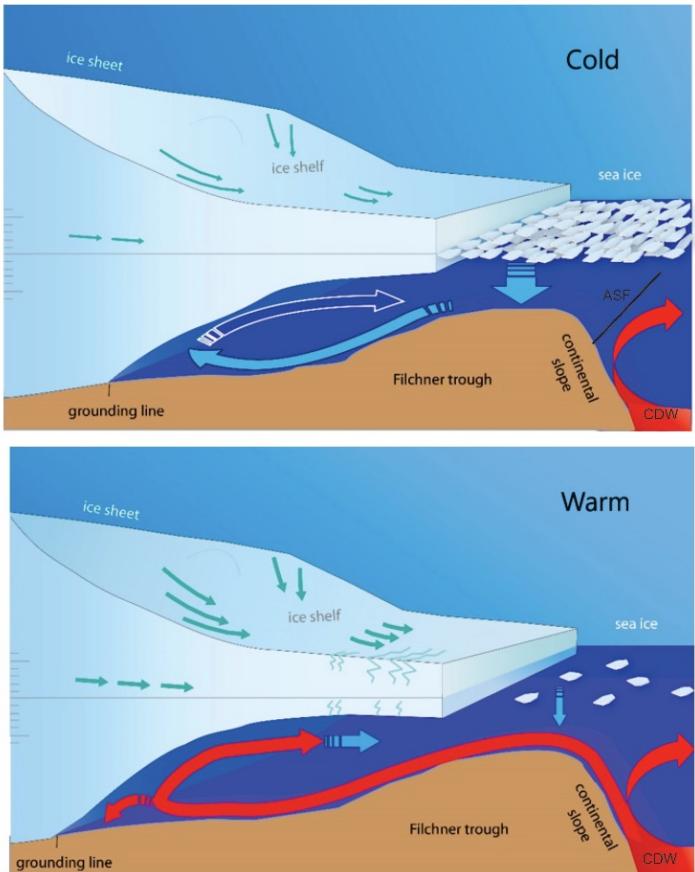
[Pattyn and Morlighem, 2020 ; DeConto and Pollard, 2016]

Tipping points In Antarctica

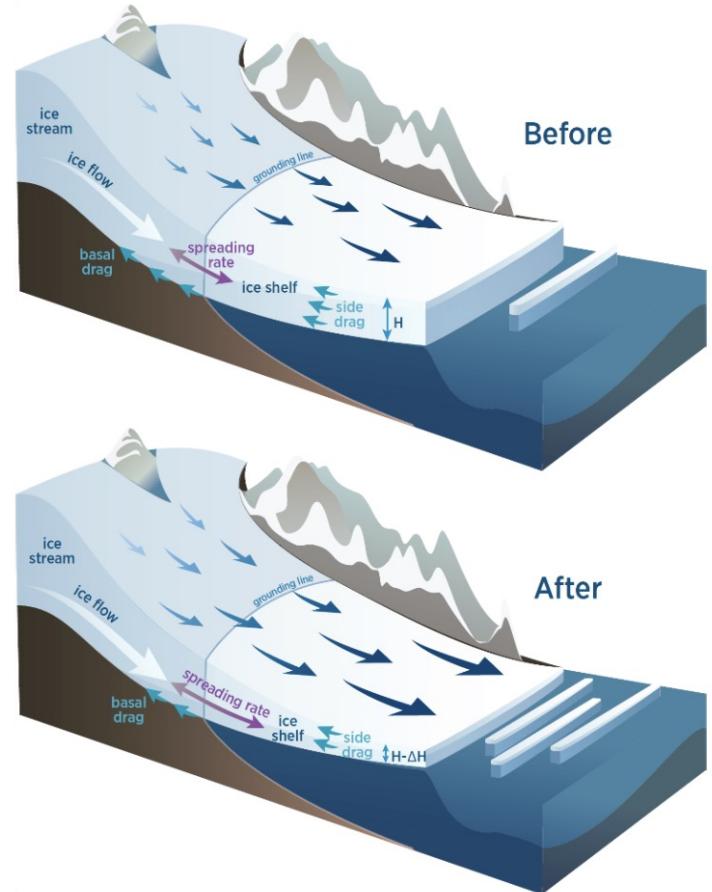


Tipping Points in Antarctic Climate Components

Ocean tipping point: cavity from cold to warm



Ice tipping point: irreversible retreat of the GL by loss of buttressing



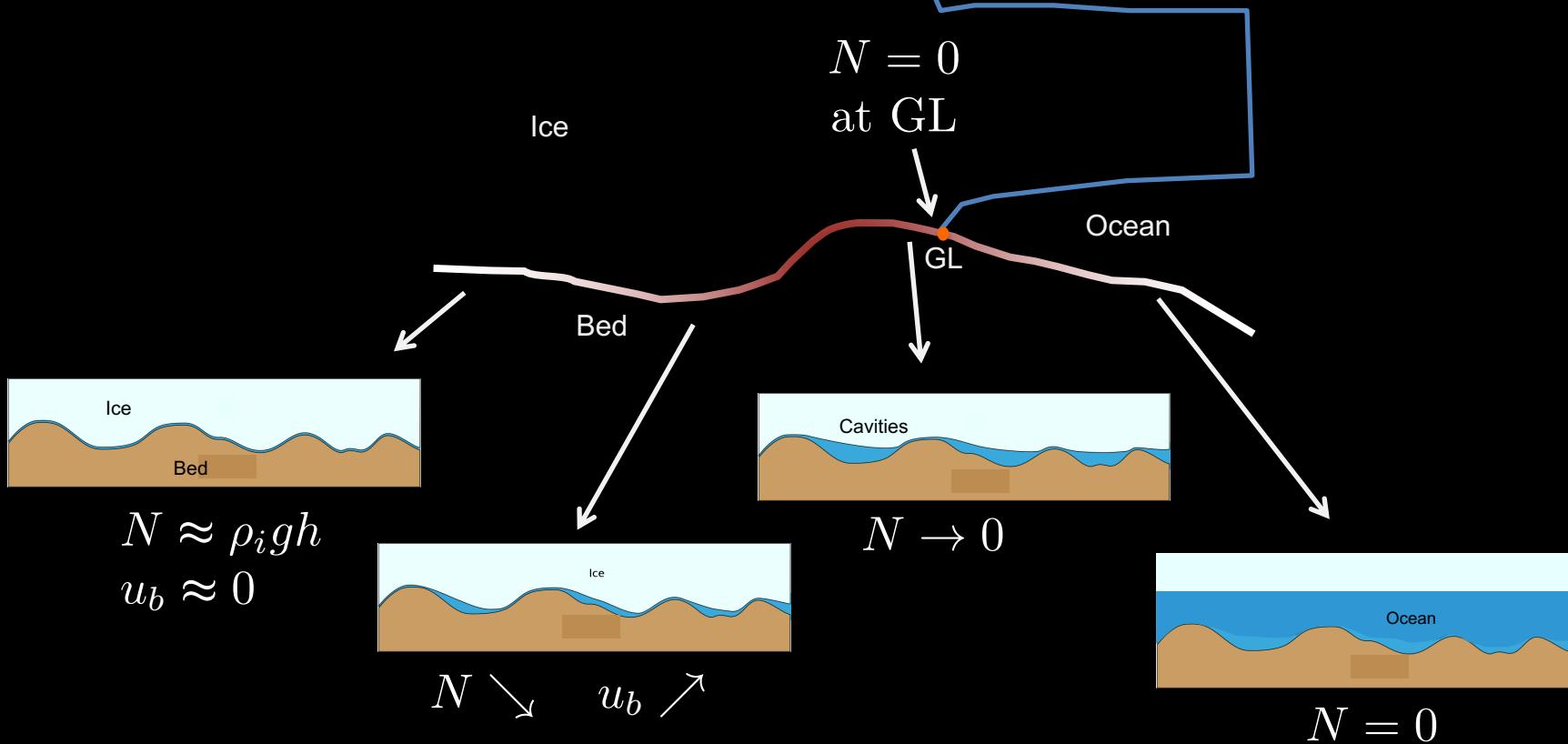
How the choice of the form
of the friction law

can affect the GL dynamics?

Types of basal conditions

$$\tau_b = f(u_b, N)$$

$$N \approx \rho_i g h - p_w$$



Which friction laws?

Weertman

$$\tau_b = C_W u_b^m$$

Coulomb

$$\tau_b = f_C N$$

Budd

$$\tau_b = C_B u_b^m N$$

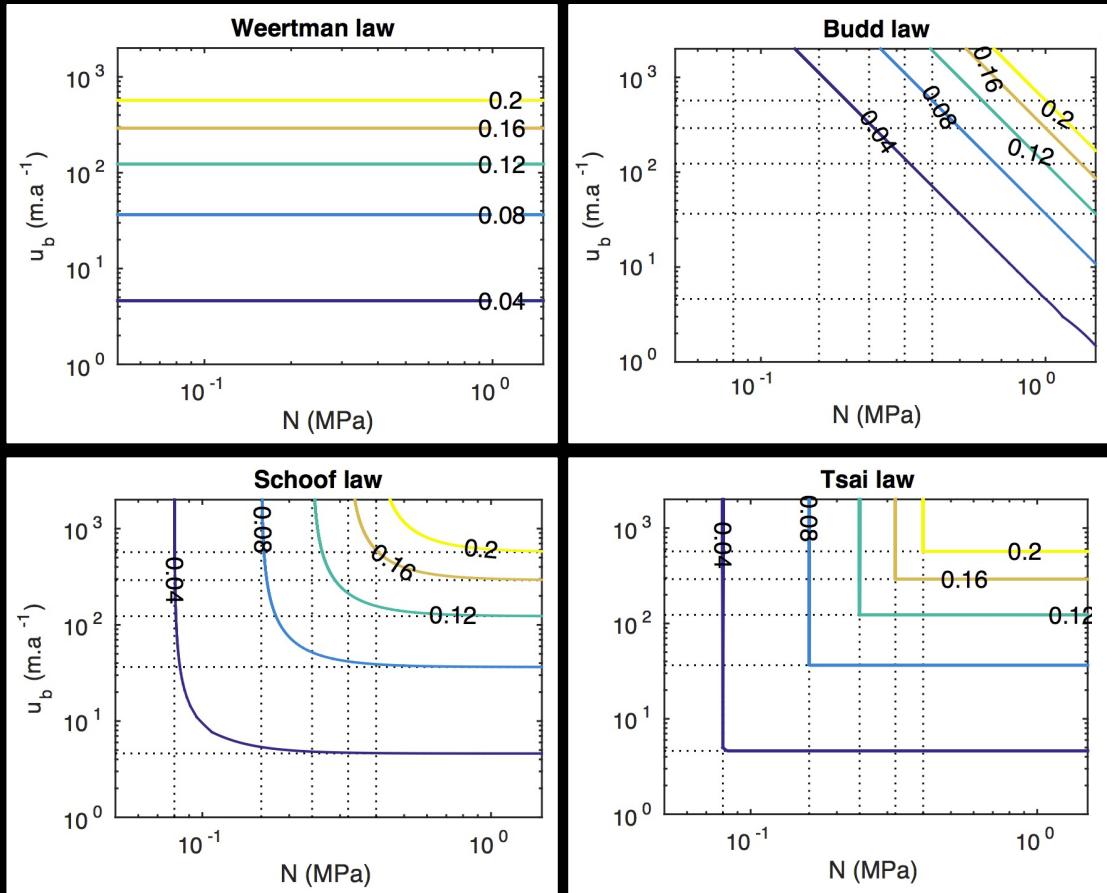
Schoof

$$\tau_b = \frac{C_S u_b^m}{\left(1 + \left(\frac{C_S}{C_{\max}}\right)^{1/m} u_b\right)^m}$$

Tsai

$$\tau_b = \min [C_W u_b^m, f_C N]$$

Isovalues of τ_b [MPa]

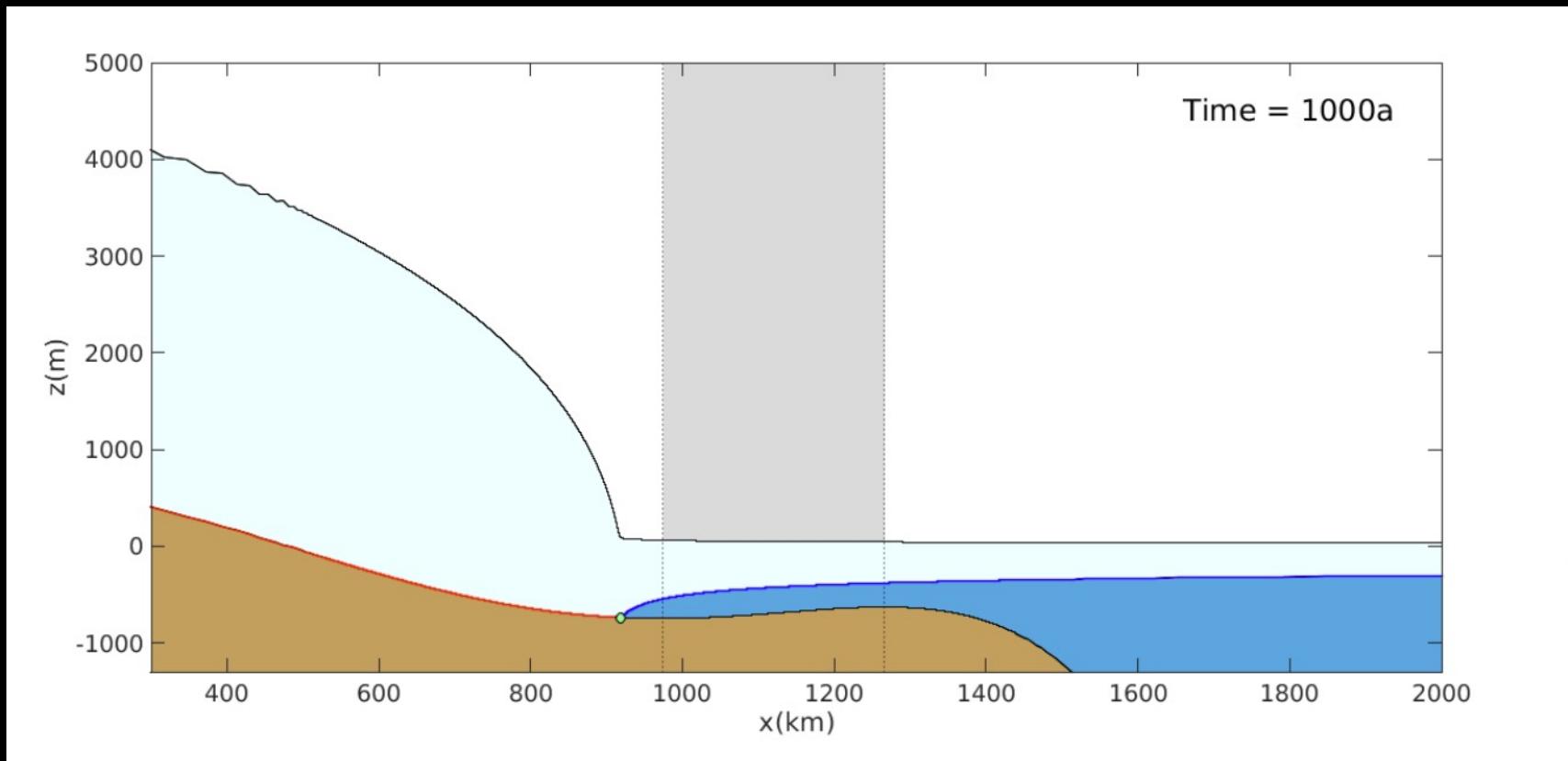


$$m = 1/3, \quad C_W = C_B = C_S, \quad C_{\max} = f_C$$

[Weertman 1957; Budd et al., 1984; Schoof 2005; Tsai et al., 2015]

[Brondex et al., 2017]

2D exp - MISMIP setup



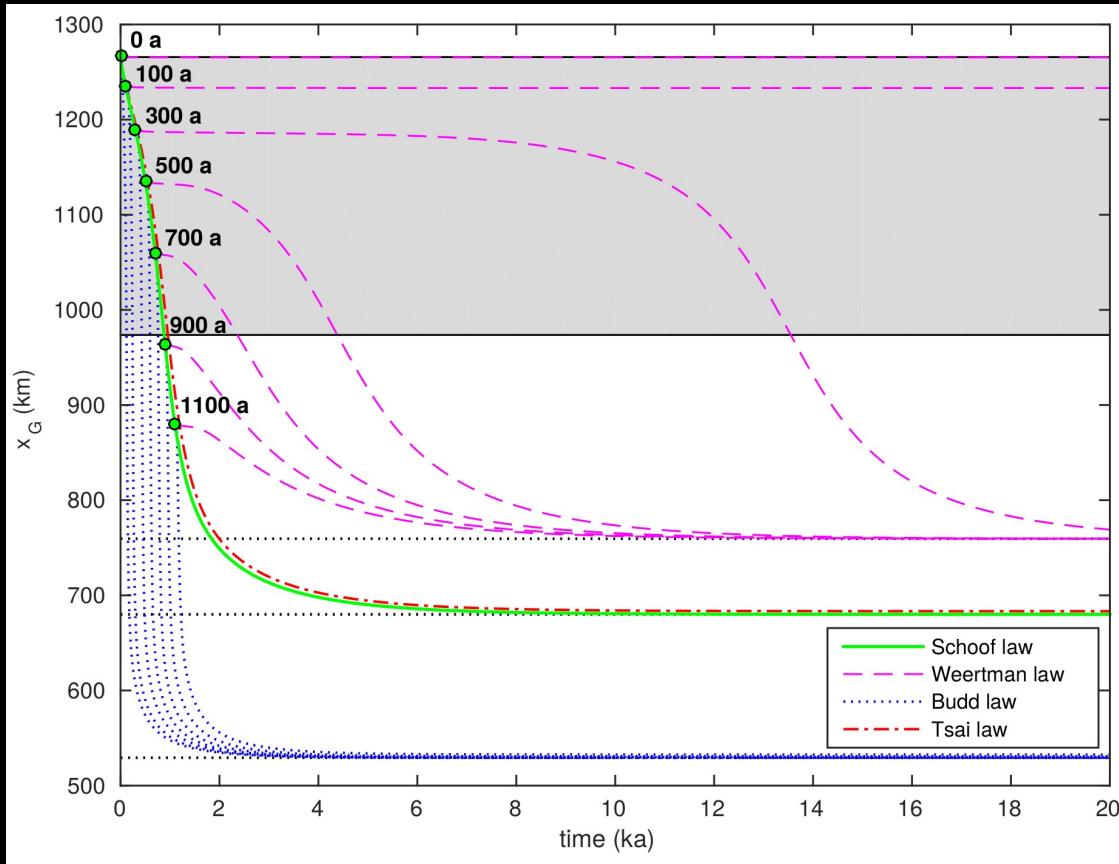
Assume $N = \rho_i gh - \rho_w g(Z_{sl} - Z_b)$

[Brondex et al., 2017]

Use Schoof as "observations"

Compare Weertman, Budd, Tsai to Schoof

2D exp - GL retreat speed



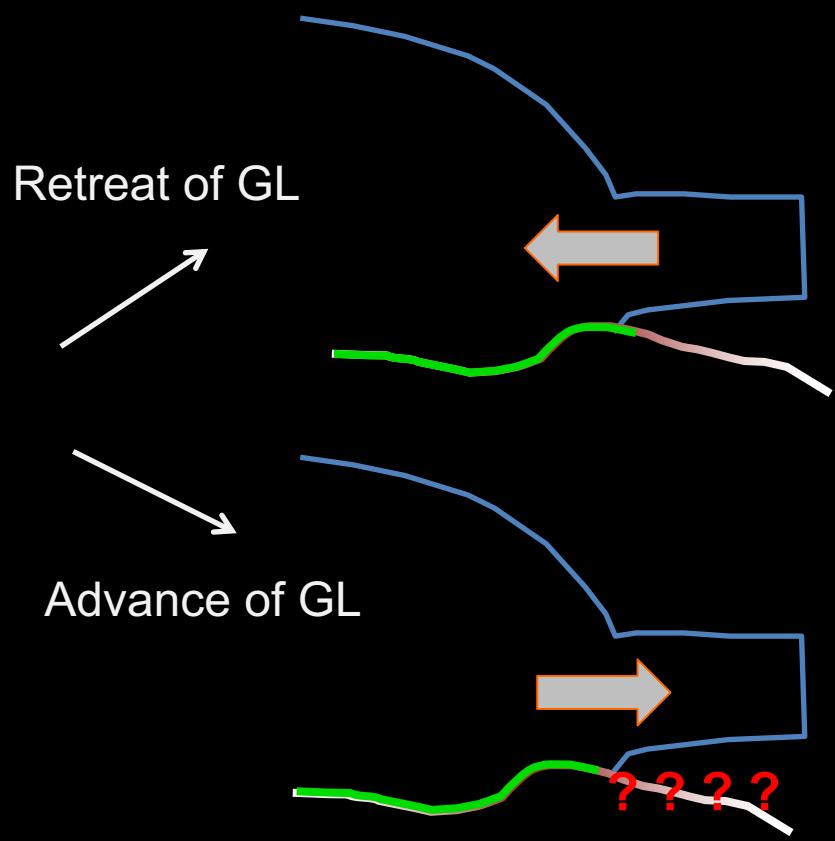
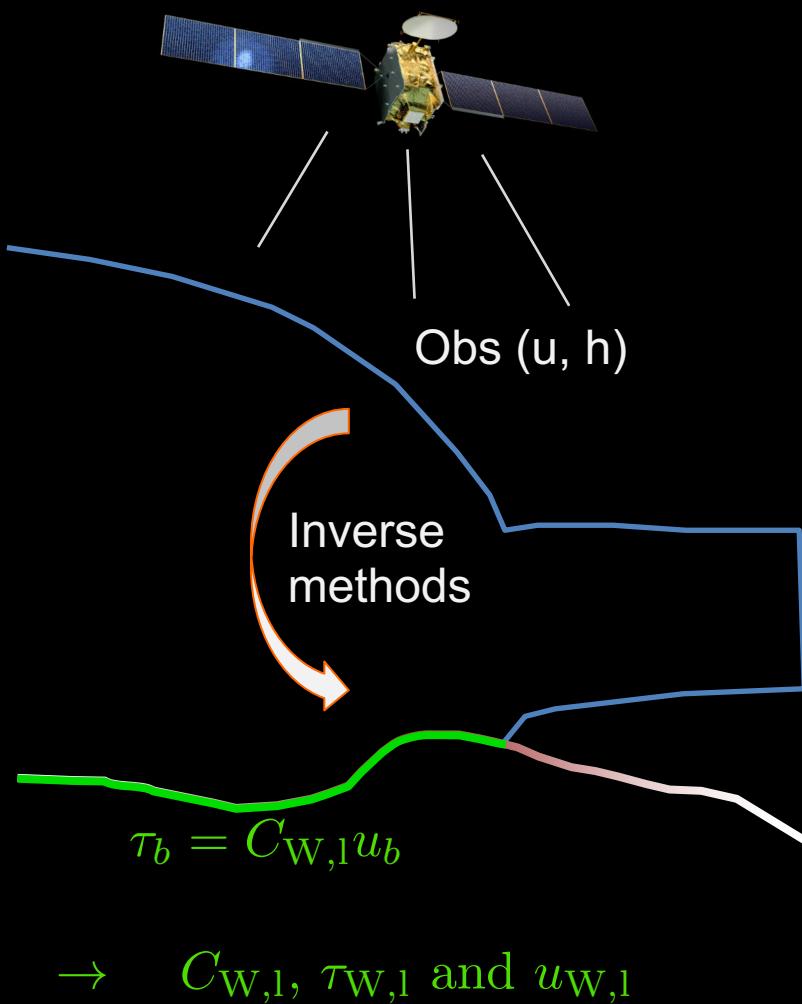
Weertman ~ 100 m/a

Schoof / Tsai
 ~ 500 m/a

Budd ~ 9000 m/a

[Brondex et al., 2017]

Problematic of inverting basal conditions (1/2)

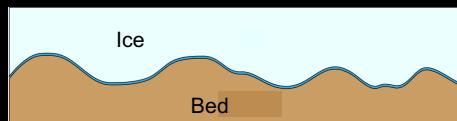


Problematic of inverting basal conditions (2/2)

$$C_S = \frac{\tau_{W,l}}{u_{W,l} \left(1 - \left(\frac{\tau_{W,l}}{C_{\max} N} \right)^{1/m} \right)^m}$$

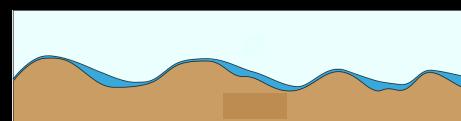
C_S defined only if $\frac{\tau_{W,l}}{N} < C_{\max}$

$$N \approx \rho_i g h$$



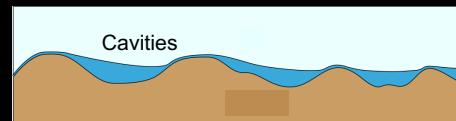
C_S

$$N \searrow$$



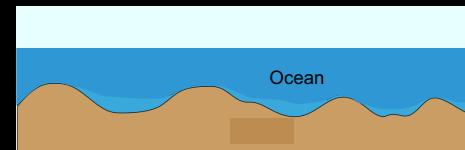
C_S

$$N \rightarrow 0$$



$\cancel{C_S}$

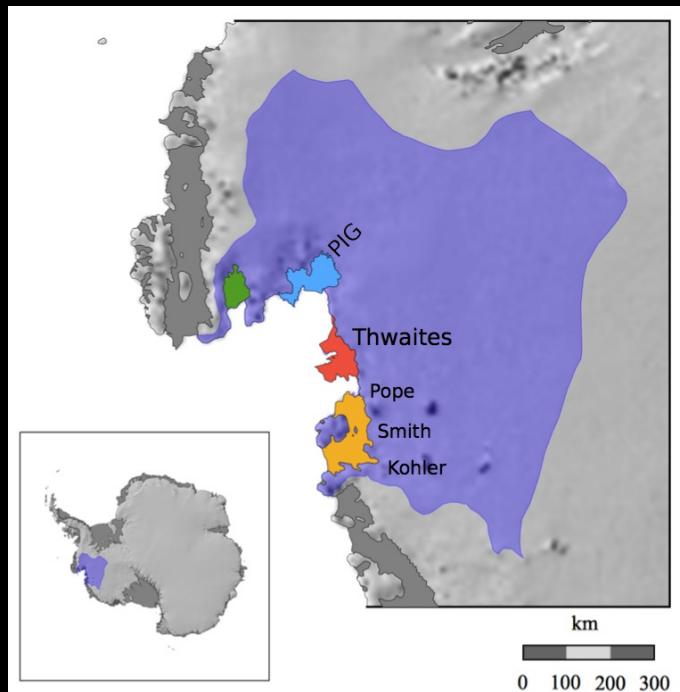
$$N = 0$$



$\cancel{C_S}$

Extrapolation
of C_S
depends on C_{\max} !

Admunsden basin

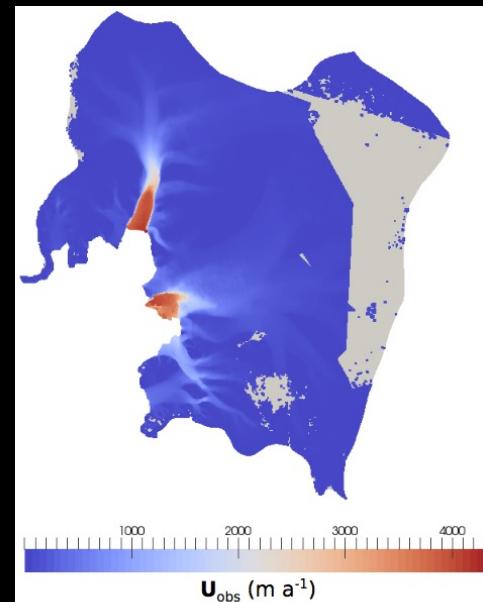


Build an initial configuration

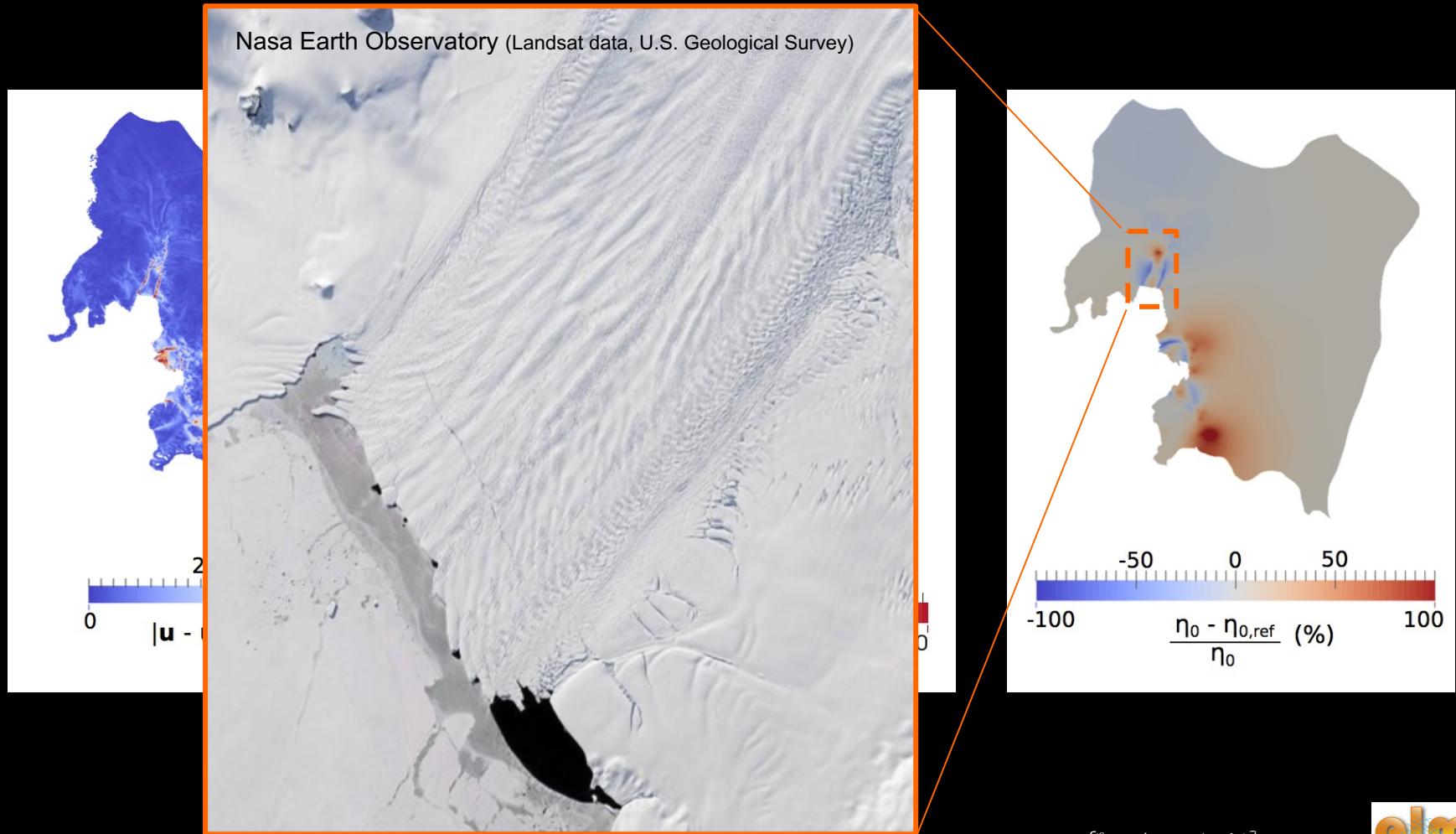
Compare Weertman and Schoof for a melt perturbation experiment (InitMIP)

Data:

- velocities [Rignot et al., 2011]
- DEMs [Fretwell et al., 2013; Millan et al., 2017]
- Temperature [Van Liefferinge and Pattyn, 2013]



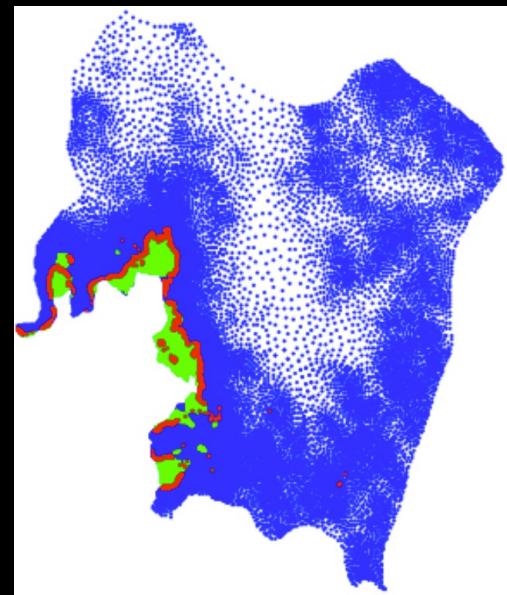
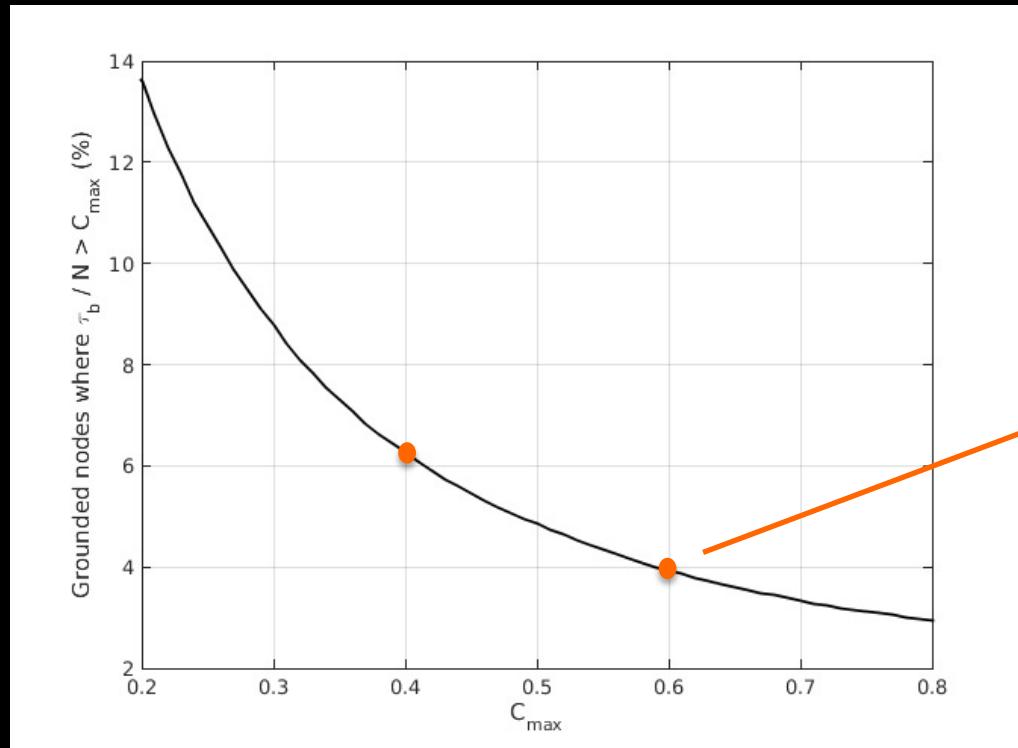
Initial configuration (inversion using linear Weertman)



[Brondex et al., 2018]

Schoof parameters

Test 2 | Ken parameters: $C_{\max} = 0.4$ $C_{\max} = 0.6$



C_S evaluated

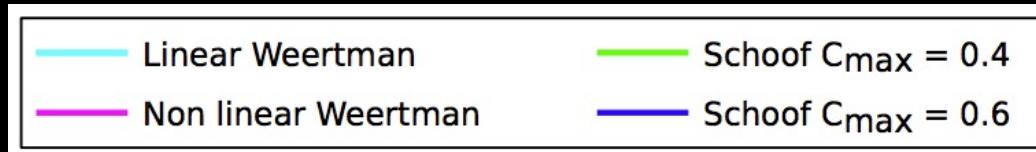
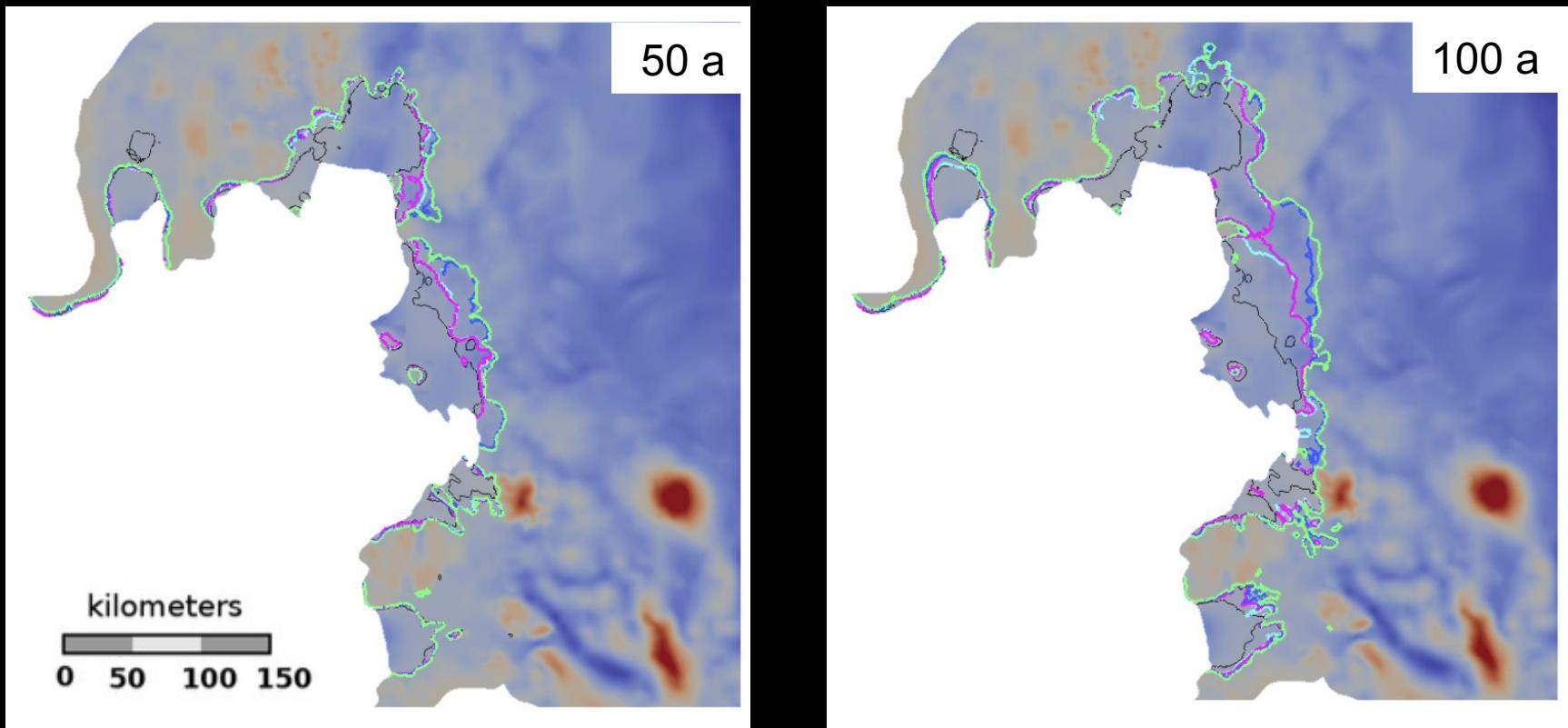
C_S extrapolated

$$C_S = \frac{\tau_{W,l}}{u_{W,l} \left(1 - \left(\frac{\tau_{W,l}}{C_{\max} N} \right)^{1/m} \right)^m}$$

$$N = \rho_i g h - \rho_w g (Z_{sl} - Z_b)$$

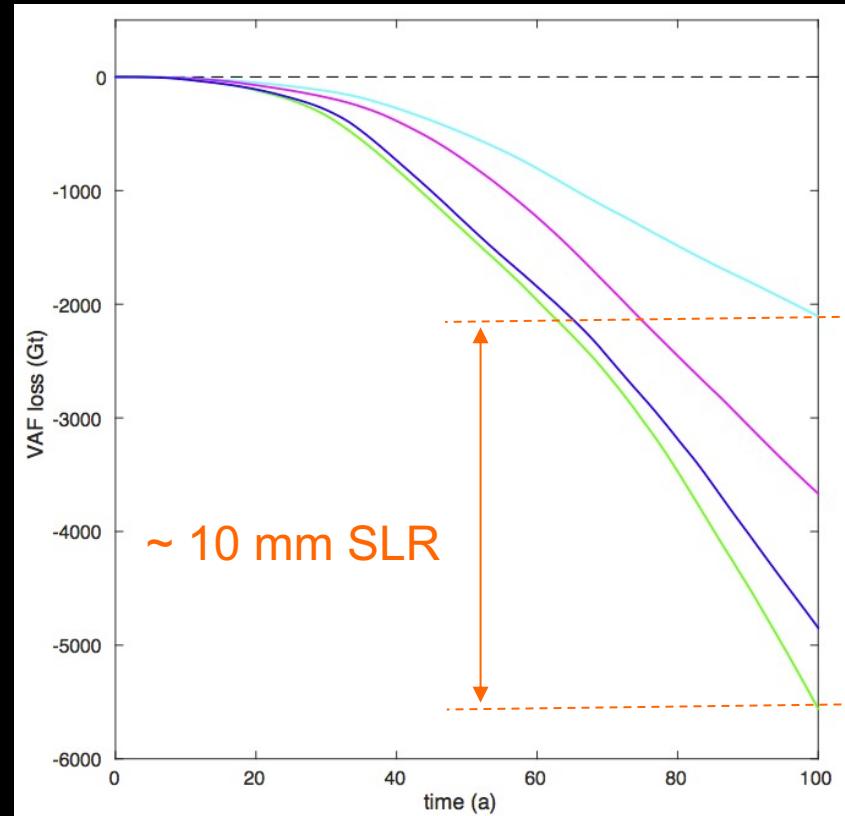
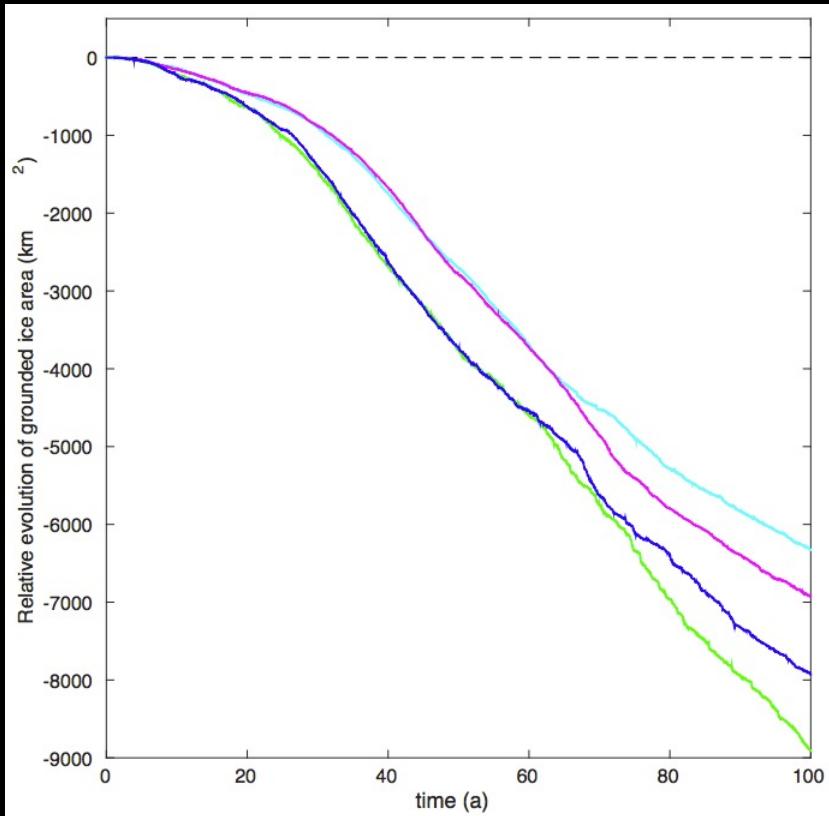
[Brondex et al., 2018]

Admunden - Results



[Brondex et al., 2018]

Admunsen - Results



Linear Weertman	Schoof C _{max} = 0.4
Non linear Weertman	Schoof C _{max} = 0.6

[Byronde et al., 2018]

What still to be done on ice sheet dynamics?

- Calving:
 - Which calving law?
 - Numerical difficulties to follow the front boundary
- MICI: need physical criteria for maximum cliff height
- Ice shelf breakoff: surface hydrology + damage

On these last points, non-continuous model (e.g. discrete particles models) might help

- Models coupling:
 - With Ocean (cavity bellow ice-shelves)
 - With atmosphere
- And always : improve numerical efficiency (A.I.?, emulators?) and resolution
- And certainly much more...

What still to be done on friction/hydrology?

- How to account for the short-term changes in water pressure within a friction law?

$$\tau_b = f(u_b, N) \rightarrow \tau_b = f(u_b, N, h) ?$$

- Reconciliate the cavity size from the FEM cavity model and the cavity size (h) from the hydrology model
- Do we really need to account for hydrology for multi-decade simulations?
- A universal friction law for both hard-bed and soft-bed?
- And certainly much more...